AP-C Objectives (from College Board Learning Objectives for AP Physics)

- 1. Current, Resistance, Power
  - a. Understand the definition of electric current and relate magnitude and direction of the current to the rate of flow of positive and negative charge.
  - b. Understand conductivity, resistivity and resistance.
    - i. Relate current and voltage for a resistor.
    - ii. Write the relationship between electric field strength and current density in a conductor and describe, in terms of the drift velocity of electrons, why such a relationship is plausible.
    - iii. Describe how the resistance of a resistor depends upon its length and cross-sectional area, and apply this result in comparing current flow in resistors of different material or geometry.
    - iv. Derive an expression for the resistance of a resistor of uniform cross-section in terms of its dimensions and the resistivity of the material from which it is constructed.
    - v. Derive and apply expressions that relate the current, voltage, and resistance to the rate at which heat is produced when current passes through a resistor.
- 2. Steady-state direct current circuits with batteries and resistors only
  - a. Understand the behavior of series and parallel combinations of resistors in order to
    - i. Analyze a circuit consisting of both series and parallel resistive elements in order to find current, voltage, resistance, and power dissipated for any/all appropriate elements.
    - ii. Design a simple circuit consisting of series and parallel elements to provide a given current through and potential difference across a specified component, and draw a schematic diagram for the circuit.
  - b. Understand the properties of ideal and real batteries in order to
    - i. Calculate the terminal voltage of a battery of specified emf and internal resistance from which a known current is flowing.
    - ii. Calcualte the rate at which a battery is supplying energy to a circuit or is being charged up by a circuit.
  - c. Apply Ohm's Law and Kirchhoff's Rules to DC circuits in order to:
    - i. Determine a single unknown current, voltage, or resistance.
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  - d. Understand the properties of voltmeters and ammeters in order to:
    - i. State whether the resistance of each type of meter is high or low.
    - ii. Identify or demonstrate correct methods of connecting meters into circuits in order to measure voltage or current.
    - iii. Assess qualitatively the effect of finite meter resistance on a circuit into which these meters are connected.
- 3. Capacitors in Circuits
  - a. Understand the initial and steady-state behavior of capacitors connected in series or in parallel in order to
    - i. Calculate the equivalent capacitance of a series or parallel combination.
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    - iii. Determine the ratio of voltages for capacitors connected in series.
    - iv. Calculate the voltage or stored charge, under steady-state conditions, for a capacitor connected to a circuit consisting of a battery and resistors.
  - b. Understand the charging or discharging of a capacitor through a resistor in order to
    - i. Calculate and interpret the time constant of the circuit.
    - ii. Sketch or identify graphs of stored charge or voltage for the capacitor or resistor.
    - iii. Write expressions to describe the time dependence of the stored charge, voltage, or current for elements in an RC circuit.
    - iv. Analyze the behavior of circuits containing several capacitors and resistors, including analyzing or sketching graphs to indicate how voltages, currents, and charges vary with time.

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Electric Current

Electric current is the flow rate of electric charge. Units are C/s, or amperes (A). Positive current flow direction is the direction of flow of positive charges, which is opposite the direction of electron flow.

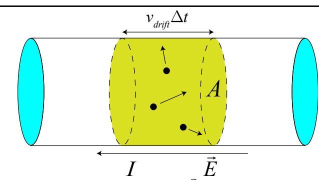
$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

Drift Velocity

In a conductor, electrons are in constant thermal motion. The net electron flow, however, is zero because the motion is random. When an electric field is applied, however, a small net flow in a direction opposite the electric field is observed. The average velocity of these electrons due to the electric field is known as the electron drift velocity (v_d).

Derivation of Current Flow

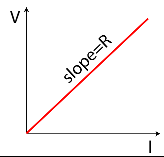
Consider a uniform conductor of cross-sectional area A. Electrons in the conductor move randomly with thermal velocities (on the order of 10^6 m/s). When an electric field is applied, however, there is some small net movement of electrons (~ .5 cm/s) opposite the direction of the electric field (v_d). If we define N as the volume density of charge carriers (electrons), the electrons contained in volume $v_d \Delta t A$ will pass the surface A in time Δt. From this, the total charge passing surface A is equal to the product of the volume passing surface A ($v_d \Delta t A$), the carrier density (N), and the charge on each charge carrier (e). Current, then, is the rate at which the charge flows past surface A, and can easily be found by dividing the charge passing A by the time interval.



$$Q = Nev_d \Delta t A \xrightarrow{I=\frac{Q}{\Delta t}} I = Nev_d A$$

Resistance

Resistance is the ratio of the potential drop across an object to the current flowing through the object. Objects which have a fixed resistance (not a function of current or potential drop) are known as ohmic materials and are said to follow Ohm's Law (an empirical law).



$$R = \frac{V}{I} \rightarrow V = IR$$

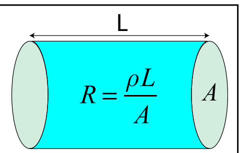
Current Density

The current density through a surface is the current per area, and is a vector quantity (**J**).

$$\vec{J} = Ne\vec{v}_d \rightarrow I = \int_S \vec{J} \bullet d\vec{A}$$

Resistance of a Wire

The resistance of a wire depends on the geometry of the wire as well as a material property known as resistivity (ρ), which relates to the ability of the material to resist the flow of electrons.



Conversion of Electric Energy to Thermal Energy

$$W = qV \rightarrow \frac{dW}{dt} = \frac{d}{dt}(qV) \xrightarrow{P=\frac{dW}{dt}} P = V\frac{dq}{dt} \xrightarrow{I=\frac{dq}{dt}}$$

$$P = IV \xrightarrow{V=IR} P = I^2R \xrightarrow{I=\frac{V}{R}} P = \frac{V^2}{R}$$

Sample Problem: Silver Wire

A silver wire with a 0.5-mm-radius cross-section is connected to the terminals of a 1-V battery. If the wire is 0.1 m long, determine the resistance of the wire, the current flowing through the wire, the drift velocity of the free electrons in the wire (assume one free electron per atom), and the average time required for electrons to pass from the negative terminal of the battery to the positive terminal. The resistivity of silver is 1.59×10^{-8} Ω•m, its molar mass is 107.9 g/mole, and its mass density is 10.5 g/cm³.

$$R = \frac{\rho L}{A} = \frac{(1.59\times10^{-8}\Omega \bullet m)(0.1m)}{\pi(.0005m)^2} = 0.00202\Omega \qquad I = \frac{V}{R} = \frac{1V}{0.00202\Omega} = 494A$$

In order to find the drift velocity, we first need to know the charge carrier density. We'll determine this by dividing Avogadro's number by the volume of a mole of silver. Then, we can find the drift velocity from our formula for current.

$$N = \frac{N_A}{V} \xrightarrow{\rho_{silver}=\frac{M}{V} \rightarrow V=\frac{M}{\rho_{silver}}} N = \frac{N_A \rho_{silver}}{M}$$

$$I = Nev_d A \rightarrow v_d = \frac{I}{NeA} \xrightarrow{N=\frac{N_A \rho_{silver}}{M}} v_d = \frac{IM}{N_A \rho_{silver} eA} \rightarrow$$

$$v_d = \frac{(494A)(0.1079 \, {}^{kg}/_{mol})}{(6.02\times10^{23} \, {}^{e}/_{mol})(10500 \, {}^{kg}/_{m^3})(1.6\times10^{-19} \, {}^{C}/_{e})(\pi\times.0005m^2)} = 0.067 \, {}^{m}/_{s}$$

And finally, knowing the drift velocity and the length, we can solve for time.

$$v_d = \frac{L}{t} \rightarrow t = \frac{L}{v_d} = \frac{0.1m}{0.067 \, {}^{m}/_{s}} = 1.49s$$

Refining Ohm's Law

$$V = IR \xrightarrow{R=\frac{\rho L}{A}} V = \frac{I\rho L}{A} \xrightarrow{E=V/L}$$

$$E = \rho\frac{I}{A} \xrightarrow{J=I/A} \vec{E} = \rho\vec{J}$$

DC Circuits with Batteries and Resistors Only

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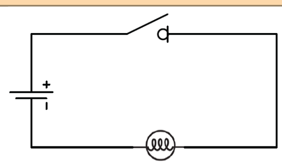
  - ● iii. Assess qualitatively the effect of finite meter resistance on a circuit into which these meters are connected.

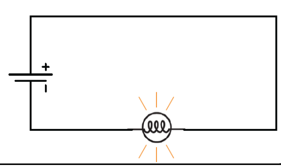
▼ Electric Circuits

Electrical circuits are closed-loop paths through which current can flow. Conventional current flows from high to low potential, though in actuality, in most cases electrons are the charge carriers, and flow from low to high potential (in the opposite direction of the current).

▼ Circuit Schematics

Three-dimensional electrical circuits are often represented in two dimensions using circuit schematics. Symbols represent the circuit elements, while lines represent wires. A source of potential difference is required for current to flow (voltaic cells, batteries, power supplies such as voltage sources or current sources). Note that current only flows in a complete path.



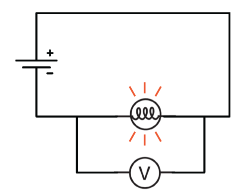


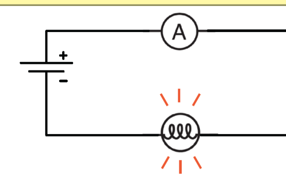
▼ Voltmeters

Voltmeters measure the potential difference between two points in a circuit. Voltmeters are connected in parallel with the element to be measured. If a voltmeter is connected correctly, you can remove it from the circuit without breaking the circuit. Voltmeters have very high resistance to minimize their effect on the circuit's performance.

▼ Ammeters

Ammeters measure the current flowing through a circuit element. Ammeters are connected in series with the circuit, so that the current to be measured flows through the ammeter. The circuit must be broken to correctly insert an ammeter. Ammeters have very low resistance to minimize the potential drop through the ammeter.

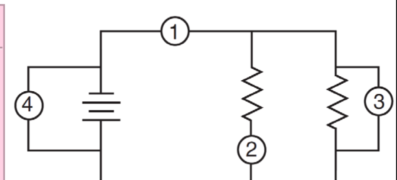




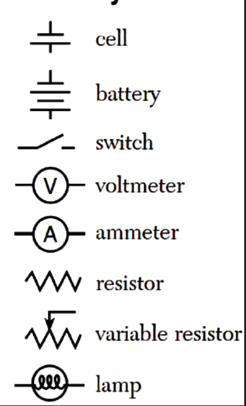
▼ Sample Problem: Ammeter and Voltmeter Placement

In the electric circuit diagram, possible locations of an ammeter and voltmeter are indicated by circles 1, 2, 3, and 4. Where should an ammeter be placed to measure the total current and where should a voltmeter be located to measure the total voltage?

Answer: Ammeter at 1 and voltmeter at 4



Circuit Symbols



▼ Series and Parallel Circuits

Series circuits have only a single current path. Removal of any circuit element causes an open circuit.

Parallel circuits have multiple current paths. Removal of a circuit element may allow other branches of the circuit to continue operating.

▼ Kirchhoff's Current Law (KCL) aka "Junction Rule"

The sum of all current entering any point in a circuit equals the sum of all current leaving any point in a circuit. This is a restatement of the law of conservation of charge.

▼ Kirchhoff's Voltage Law (KVL) aka "Loop Rule"

The sum of all the potential drops in any closed loop of a circuit has to equal zero. This is a restatement of the law of conservation of energy.

▼ Analysis of DC Circuits

A VIRP table is a useful tool for analyzing circuits. More extensive circuit analysis practice and explanations are available on the APlusPhysics website directly.

▼ Equivalent Resistance

Resistors in series and parallel configurations may be replaced by a single resistor of equivalent total resistance. This is especially useful for circuit analysis, even when not practically advantageous.

$$R_\Sigma = R_1 + R_2 + R_3 + \ldots \rightarrow R_\Sigma = \sum_i R_i$$

$$\frac{1}{R_\parallel} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \rightarrow \frac{1}{R_\parallel} = \sum_i \frac{1}{R_i}$$

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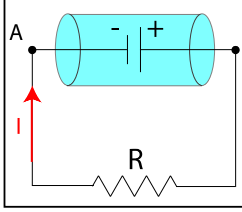
- ▼ a. Understand the properties of ideal and real batteries in order to
  - i. Calculate the terminal voltage of a battery of specified emf and internal resistance from which a known current is flowing.
  - ii. Calcualte the rate at which a battery is supplying energy to a circuit or is being charged up by a circuit.

▼ Batteries

A cell or battery (a combination of cells) provides a potential difference, oftentimes referred to as an electromotive force or emf ($\mathcal{E}$). Note that an emf is not a force, but a potential difference, despite the confusing name. The emf of a cell or battery is the maximum possible potential difference that device can provide. A battery can be thought of as a pump for charge, raising it from a lower potential to a higher potential.

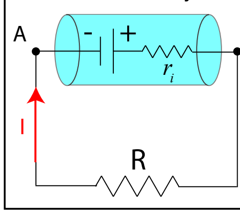
Ideal batteries have no resistance. Real batteries, however, have some amount of resistance to the flow of charge within the battery itself, known as the internal resistance of the battery (r_i). In an ideal battery, the terminal voltage (V_T) is equal to its emf. In a real battery, however, the terminal voltage (V_T) is slightly lower than the batteries emf as some voltage is dropped across the battery's internal resistance.

Ideal Battery



$$V_{battery} = \Delta V = V_B - V_A \rightarrow$$
$$V_{battery} = \varepsilon = V_T$$

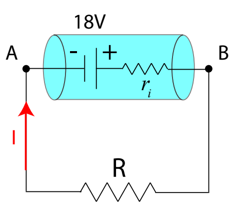
Real Battery



$$V_{battery} = \Delta V = V_B - V_A = IR \rightarrow$$
$$V_{battery} = \varepsilon - Ir_i = V_T$$

▼ Sample Question: Internal Resistance of a Battery

The terminal voltage of a real battery is 15 volts. If the battery has an emf of 18 volts and supplies 10 watts of power to resistor R, find the value of R and r_i.



	V	I	R	P
r_i	3	.67	4.5	2
R	**15**	.67	22.5	**10**
Total	**18**	.67	27	12

RC Circuits - Steady State

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   - a. Understand the initial and steady-state behavior of capacitors connected in series or in parallel in order to
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     - iii. Determine the ratio of voltages for capacitors connected in series.
     - iv. Calculate the voltage or stored charge, under steady-state conditions, for a capacitor connected to a circuit consisting of a battery and resistors.
   - b. Understand the charging or discharging of a capacitor through a resistor in order to
     - i. Calculate and interpret the time constant of the circuit.
     - ii. Sketch or identify graphs of stored charge or voltage for the capacitor or resistor.
     - iii. Write expressions to describe the time dependence of the stored charge, voltage, or current for elements in an RC circuit.
     - iv. Analyze the behavior of circuits containing several capacitors and resistors, including analyzing or sketching graphs to indicate how voltages, currents, and charges vary with time.

What is a Capacitor?

A capacitor is a device which stores electrical energy. Consisting of two conducting plates separated by an insulator, a capacitor holds opposite charges on each plate with a potential difference across the plates.

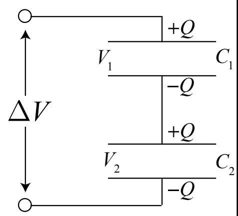
Capacitance

Capacitance is the ratio of the charge separated on the plates of the capacitor to the potential difference between the plates. Units of capacitance are coulombs/volt, or farads.

$$C=\frac{Q}{V}$$

Capacitors in Series

Capacitors in series have the same charge because each plate must obtain charge from the next plate due to conservation of charge.



$$C_{eq}=\frac{Q}{V}=\frac{Q}{V_1+V_2}=\frac{Q}{\frac{Q}{C_1}+\frac{Q}{C_2}}=\frac{1}{\frac{1}{C_1}+\frac{1}{C_2}}\rightarrow$$

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\ldots$$

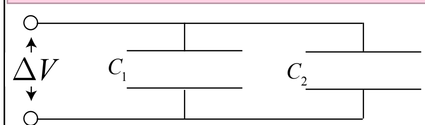
$$\frac{V_2}{V_1}=\frac{\frac{Q_2}{C_2}}{\frac{Q_1}{C_1}}\xrightarrow{Q_1=Q_2}\frac{V_2}{V_1}=\frac{C_1}{C_2}$$

Capacitors in Circuits

From a DC perspective, when a capacitor is uncharged it acts like a wire. Once it is charged, it acts like an open circuit.

Capacitors in Parallel

Capacitors in parallel have the same voltage across their plates due to conservation of energy.



$$C_{eq}=\frac{Q}{V}=\frac{Q_1+Q_2}{V}=\frac{Q_1}{V}+\frac{Q_2}{V}=C_1+C_2\rightarrow$$

$$C_{eq}=C_1+C_2+\ldots$$

$$\frac{Q_1}{Q_2}=\frac{C_1V_1}{C_2V_2}\rightarrow\frac{Q_1}{Q_2}=\frac{C_1}{C_2}$$

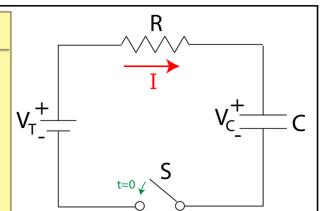
Charging an RC Circuit

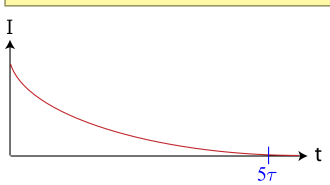
At time t=0, the switch is closed and the capacitor initially acts like a wire. Applying KVL:

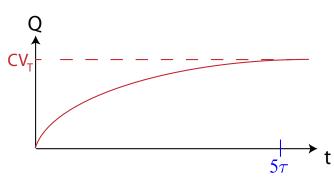
$$-V_T+IR+V_C=0\xrightarrow[V_C=\frac{Q}{C}]{C=\frac{Q}{V_C}}-V_T+IR+\frac{Q}{V_C}=0\xrightarrow{Q=0\ @t=0}V_T=IR$$

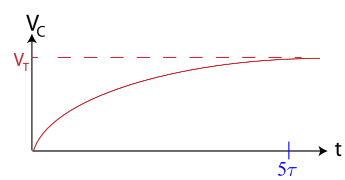
After a long time (~ 5 time constants, or 5τ) the capacitor acts like an open.

$$-V_T+IR+V_C=0\xrightarrow{I=0}V_C=V_T$$









The Time Constant (τ)

The time constant in an RC circuit (τ=RC) indicates the time at which the quantity under observation has achieved ($1-e^{-1}$), or 63%, of its final value. By 5 time constants ($5\tau=5\times RC$), the quantity under observation is within 1 percent of its final value.

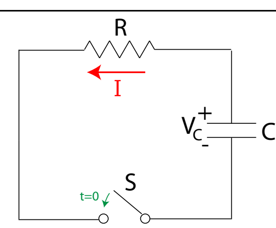
Discharging an RC Circuit

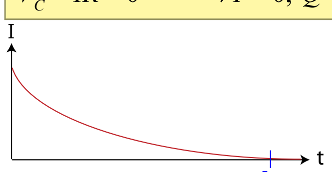
Assume capacitor is fully charged. At time t=0, the switch is closed and the capacitor initially acts like a source of potential difference (a battery). Applying KVL:

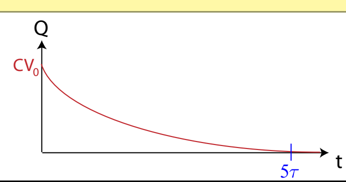
$$V_C-IR=0\rightarrow I=\frac{V_C}{R},\ Q=CV_C$$

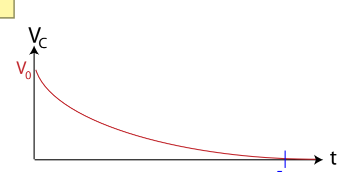
After a long time (~ 5 time constants, or 5τ) the capacitor acts like a wire.

$$V_C-IR=0\xrightarrow{V_c=0}I=0,\ Q=0$$









RC Circuits - Transient Analysis

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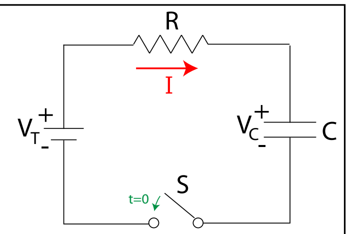
▼ Transient Analysis of RC Circuits

Analyzing RC circuits when capacitors are completed charged or uncharged is relatively straightforward, but a deeper look at the electrical characteristics of these circuits as the capacitor is charging or discharging is known as transient analysis.

▼ Charging an RC Circuit

At time t=0, the switch is closed and the capacitor initially acts like a wire.



$$-V_T + IR + V_C = 0 \xrightarrow[V=\frac{Q}{C}]{C=\frac{Q}{V}} V_T = IR + \frac{Q}{C} \xrightarrow{I=\frac{dQ}{dt}} V_T = R\frac{dQ}{dt} + \frac{Q}{C} \to \frac{V_T}{R} = \frac{dQ}{dt} + \frac{Q}{RC} \to$$

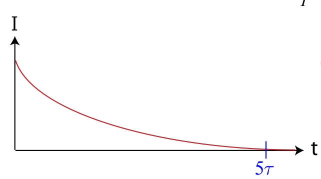
$$\frac{dQ}{dt} = \frac{V_T}{R} - \frac{Q}{RC} \to \frac{dQ}{dt} = \frac{CV_T - Q}{RC} \to \frac{dQ}{CV_T - Q} = \frac{dt}{RC} \to \int_{Q=0}^{Q}\frac{dQ}{CV_T-Q} = \int_{t=0}^{t}\frac{dt}{RC} \to$$

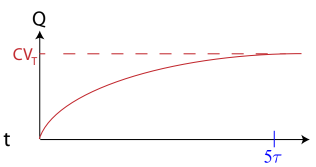
$$-\ln(CV_T - Q)\Big|_0^Q = \frac{t}{RC}\Big|_0^t \to -\ln(CV_T-Q) - (-\ln(CV_T)) = \frac{t}{RC} \to \ln(CV_T - Q) - \ln(CV_T) = -\frac{t}{RC} \to$$

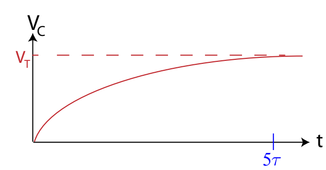
$$\ln\left(1-\frac{Q}{CV_T}\right) = -\frac{t}{RC} \xrightarrow{e^{\ln x}=x} e^{\ln\left(1-\frac{Q}{CV_T}\right)} = e^{-\frac{t}{RC}} \to 1-\frac{Q}{CV_T} = e^{-\frac{t}{RC}} \to Q = CV_T(1-e^{-\frac{t}{RC}})$$

$$V_C = \frac{Q}{C} \to V_C = V_T(1-e^{-\frac{t}{RC}})$$

$$I = \frac{dQ}{dt} \to I = \frac{V_T}{R}e^{-\frac{t}{RC}}$$

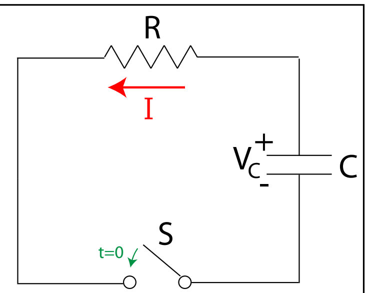






▼ Discharging an RC Circuit

At time t=0, the switch is closed and the capacitor discharges through the resistor.

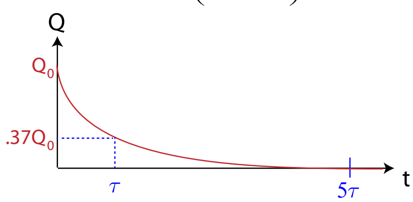


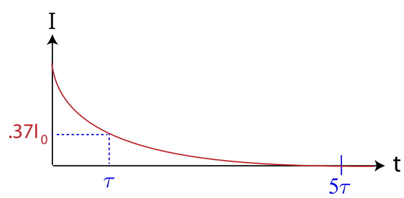
$$-V_C + IR = 0 \to V_C = IR \xrightarrow[V=\frac{Q}{C}]{C=\frac{Q}{V}} \frac{Q}{C} = IR \xrightarrow{I=-\frac{dQ}{dt}} \frac{Q}{C} = -R\frac{dQ}{dt} \to \frac{dQ}{Q} = -\frac{dt}{RC} \to$$

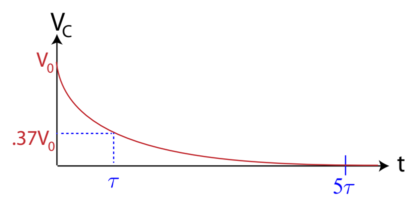
$$\int_{Q_0}^{Q}\frac{dQ}{Q} = -\int_{t=0}^{t}\frac{dt}{RC} \to \ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC} \to \frac{Q}{Q_0} = e^{-\frac{t}{RC}} \to Q = Q_0 e^{-\frac{t}{RC}}$$

$$V_C = \frac{Q}{C} = \frac{Q_0}{C}e^{-\frac{t}{RC}} \xrightarrow{V_0=\frac{Q_0}{C}} V_C = V_0 e^{-\frac{t}{RC}}$$

$$I = -\frac{dQ}{dt} = -\frac{d}{dt}\left(Q_0 e^{-\frac{t}{RC}}\right) = \frac{Q_0}{RC}e^{-\frac{t}{RC}} \xrightarrow{I_0=\frac{V_0}{R}=\frac{Q_0}{RC}} I = I_0 e^{-\frac{t}{RC}}$$







▼ The Time Constant (τ)

The time constant in an RC circuit (τ=RC) indicates the time at which the quantity under observation has achieved ($1-e^{-1}$), or 63%, of its final value. By 5 time constants (5τ=5×RC), the quantity under observation is within 1 percent of its final value.