AP-C Objectives (from College Board Learning Objectives for AP Physics)

- 1. Newton's 1st Law
  - a. Students should be able to analyze situations in which a particle remains at rest, or moves with constant velocity, under the influence of several forces (static equilibrium, N1).
- 2. Newton's 2nd Law
  - a. Students should understand the relation between the force that acts on an object and the resulting change in the object's velocity, so they can:
    - i. Calculate, for an object moving in one dimension, the velocity change that results when a constant force F acts over a specified time interval.
    - ii. Calculate, for an object moving in one dimension, the velocity change that results when a force **F**(**t**) acts over a specified time interval.
    - iii. Determine, for an object moving in a plane whose velocity vector undergoes a specified change over a specified time interval, the average force that acted on the object.
  - b. Students should understand how Newton's Second Law, $\sum F = F_{net} = ma$ applies to an object subject to forces such as gravity, the pull of strings, or contact forces, so they can:
    - i. Draw a well-labeled, free-body diagram showing all real forces that act on the object.
    - ii. Write down the vector equation that results from applying Newton's Second Law to the object, and take components of this equation along appropriate axes.
  - c. Students should be able to analyze situations in which an object moves with specified acceleration under the influence of one or more forces so they can determine the magnitude and direction of the net force, or of one of the forces that makes up the net force, such as motion up or down with constant acceleration.
  - d. Students should understand the significance of the coefficient of friction, so they can:
    - i. Write down the relationship between the normal and frictional forces on a surface.
    - ii. Analyze situations in which an object moves along a rough inclined plane or horizontal surface.
    - iii. Analyze under what circumstances an object will start to slip, or to calculate the magnitude of the force of static friction.
  - e. Students should understand the effect of drag forces on the motion of an object, so they can:
    - i. Find the terminal velocity of an object moving vertically under the influence of a retarding force dependent on velocity.
    - ii. Describe qualitatively, with the aid of graphs, the acceleration, velocity, and displacement of such a particle when it is released from rest or is projected vertically with specified initial velocity.
    - iii. Use Newton's Second Law to write a differential equation for the velocity of the object as a function of time.
    - iv. Use the method of separation of variables to derive the equation for the velocity as a function of time from the differential equation that follows from Newton's Second Law.
    - v. Derive an expression for the acceleration as a function of time for an object falling under the influence of drag forces.
- 3. Newton's 3rd Law
  - a. Students should understand Newton's Third Law so that, for a given system, they can identify the force pairs and the objects on which they act, and state the magnitude and direction of each force.
  - b. Students should be able to apply Newton's Third Law in analyzing the force of contact between two objects that accelerate together along a horizontal or vertical line, or between two surfaces that slide across one another.
  - c. Students should know that the tension is constant in a light string that passes over a massless pulley and should be able to use this fact in analyzing the motion of a system of two objects joined by a string.
  - d. Students should be able to solve problems in which application of Newton's laws leads to two or three simultaneous linear equations involving unknown forces or accelerations.

AP-C Objectives (Newton's 1st Law — Static Equilibrium)

1. Students should be able to analyze situations in which a particle remains at rest, or moves with constant velocity, under the influence of several forces (static equilibrium, N1).

Newton's 1st Law: An object at rest tends to remain at rest, and an object in motion tends to remain in motion with constant velocity unless acted upon by a net force.

Inertia: An object's resistance to being accelerated. The measure of an object's inertia is mass (kg).

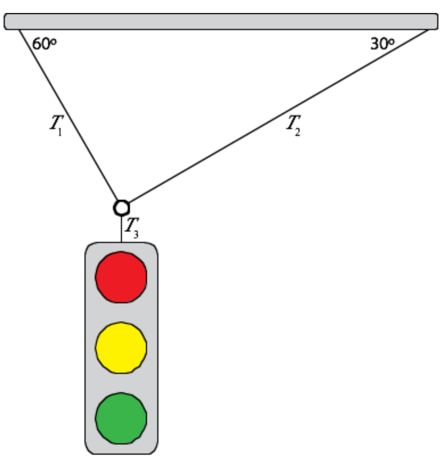
Put another way, a change in velocity (aka acceleration) only occurs when a net (unbalanced) force is applied.

▼ Inertial Reference Frames

An inertial reference frame is a reference frame in which the acceleration is zero with respect to the frame of reference, or, in a bit of an over-simplification, inertial reference frames are not accelerating. Newton's Laws of Motion only hold in inertial reference frames.

▼ Example: Suspended Traffic Light

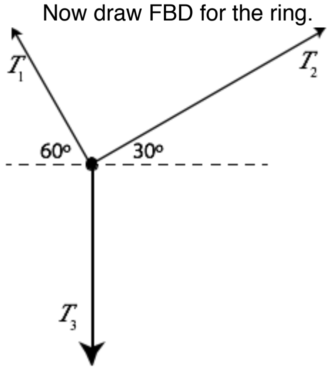
A 10-kg traffic light is suspended from a beam as shown below. Find the tension in each of the three cables supporting the traffic light.



FBD for traffic light

$$F_{net_y} = T_3 - mg = 0 \rightarrow$$
$$T_3 = mg = 10kg \times 9.8 \tfrac{m}{s^2} = 98N$$

Now draw FBD for the ring.



$$F_{net_x} = -T_1 \cos 60° + T_2 \cos 30° = 0 \rightarrow T_1 = \frac{T_2 \cos 30°}{\cos 60°}$$
$$F_{net_y} = T_1 \sin 60° + T_2 \sin 30° - mg = 0 \rightarrow$$
$$\frac{T_2 \cos 30°}{\cos 60°} \sin 60° + T_2 \sin 30° = mg \rightarrow 1.5T_2 + 0.5T_2 = mg \rightarrow T_2 = \frac{mg}{2} = 49N$$
$$T_1 = \frac{T_2 \cos 30°}{\cos 60°} = \frac{\frac{mg}{2} \cos 30°}{\cos 60°} = 0.866mg = 85N$$

AP-C Objectives (Newton's 2nd Law)

▼ 1. Newton's 2nd Law

▼ a. Students should understand the relation between the force that acts on an object and the resulting change in the object's velocity, so they can:

● i. Calculate, for an object moving in one dimension, the velocity change that results when a constant force F acts over a specified time interval.

● ii. Calculate, for an object moving in one dimension, the velocity change that results when a force **F**(**t**) acts over a specified time interval (see Work-Energy Theorem and Impulse-Momentum Theorem).

● iii. Determine, for an object moving in a plane whose velocity vector undergoes a specified change over a specified time interval, the average force that acted on the object.

▼ b. Students should understand how Newton's Second Law, $\sum F = F_{net} = ma$ applies to an object subject to forces such as gravity, the pull of strings, or contact forces, so they can:

● i. Draw a well-labeled, free-body diagram showing all real forces that act on the object.

● ii. Write down the vector equation that results from applying Newton's Second Law to the object, and take components of this equation along appropriate axes.

● c. Students should be able to analyze situations in which an object moves with specified acceleration under the influence of one or more forces so they can determine the magnitude and direction of the net force, or of one of the forces that makes up the net force, such as motion up or down with constant acceleration.

▼ d. Students should understand the significance of the coefficient of friction, so they can:

● i. Write down the relationship between the normal and frictional forces on a surface.

● ii. Analyze situations in which an object moves along a rough inclined plane or horizontal surface.

● iii. Analyze under what circumstances an object will start to slip, or to calculate the magnitude of the force of static friction.

▼ e. Students should understand the effect of drag forces on the motion of an object, so they can:

● i. Find the terminal velocity of an object moving vertically under the influence of a retarding force dependent on velocity.

● ii. Describe qualitatively, with the aid of graphs, the acceleration, velocity, and displacement of such a particle when it is released from rest or is projected vertically with specified initial velocity.

● iii. Use Newton's Second Law to write a differential equation for the velocity of the object as a function of time.

● iv. Use the method of separation of variables to derive the equation for the velocity as a function of time from the differential equation that follows from Newton's Second Law.

● v. Derive an expression for the acceleration as a function of time for an object falling under the influence of drag forces.

Free Body Diagrams

AP-C Objectives (Free Body Diagrams)

1. Draw a well-labeled, free-body diagram showing all real forces that act on the object.
2. Write down the vector equation that results from applying Newton's Second Law to the object, and take components of this equation along appropriate axes.
3. Students should be able to analyze situations in which an object moves with specified acceleration under the influence of one or more forces so they can determine the magnitude and direction of the net force, or of one of the forces that makes up the net force, such as motion up or down with constant acceleration.

▼ Free Body Diagrams

Free Body Diagrams (FBDs) are representations of an object or system drawn free from its surrounding, but showing all forces which affect the object or system.

1. Represent the system as a box (sliding) or circle (rolling).
2. Draw x- and y-axes perpendicular and parallel to the suface or direction of intended motion.
3. Draw all forces acting upon the object. For forces acting on the center of the object, draw them from the center of the box/circle. For forces acting at a distance from the object's center of mass, represent them acting at their point of contact. Label the force vectors.

▼ Pseudo Free Body Diagrams

For any forces that do not line up with the axes as drawn, create a new set of axes. Re-draw your free body diagram, this time replacing any forces that don't line up with the axes with their components such that the components and forces now all line up with the axes. Label all forces and components.

Note: Do not skip directly to this step, as you may lose points according to the AP exam grading scheme.

▼ Applying Newton's 2nd Law

Using the pseudo-FBD, write Newton's 2nd Law Equations for all axes.

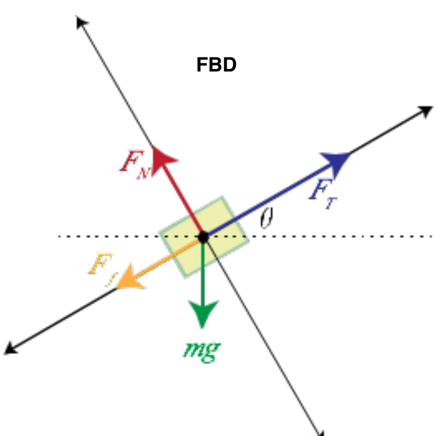
Solve resulting equations for any unknowns.

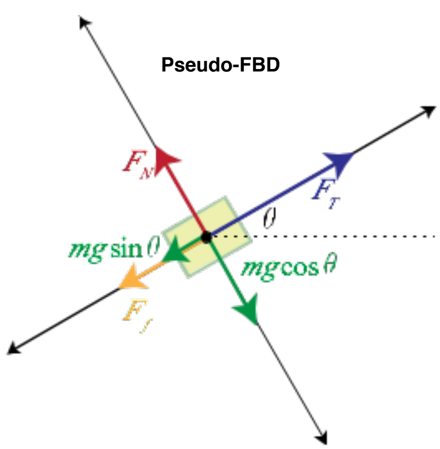
Check your answers.

$$F_{net_x} = ma_x$$
$$F_{net_y} = ma_y$$
$$F_{net_z} = ma_z$$

▼ Example

A boy pulls a sled up a 30° incline using a rope. Draw the free body diagram and pseudo-FBD.





$$\boxed{\begin{aligned} F_{net_x} &= F_T - mg\sin\theta - F_f = ma_x \\ F_{net_y} &= F_N - mg\cos\theta = ma_y = 0 \end{aligned}}$$

AP-C Objectives (Friction)

1. Students should understand the significance of the coefficient of friction, so they can:
   - a. Write down the relationship between the normal and frictional forces on a surface.
   - b. Analyze situations in which an object moves along a rough inclined plane or horizontal surface.
   - c. Analyze under what circumstances an object will start to slip, or to calculate the magnitude of the force of static friction.

Friction

The frictional force is a force exerted between two surfaces that acts parallel to the surfaces. It can be traced back to electrical forces at the molecular level. Static friction occurs for two surfaces experiencing a sliding stress but not actually moving. Kinetic friction occurs for two surfaces which do slide over each other. The coefficient of friction provides a relationship between the normal force and the frictional force.

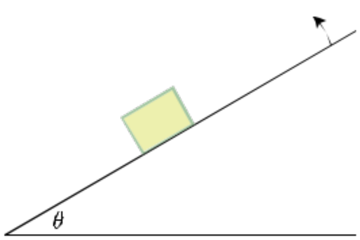
Coefficient of Friction (μ)

$$\mu = \frac{F_f}{F_N}$$

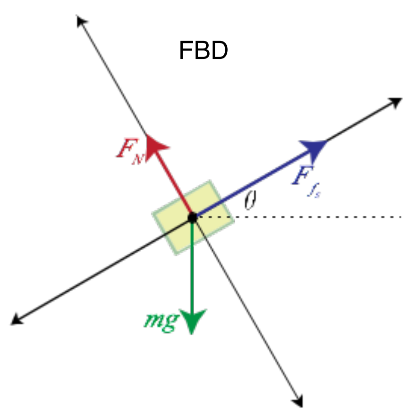
Use μ_s for static friction, and μ_k for kinetic friction.

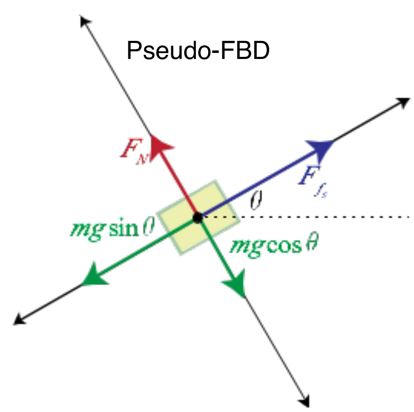
Example: Angle of Repose

A block is placed on a ramp with an unknown coefficient of friction. The angle of the ramp is increased until the block just begins to slide. Find the coefficient of static friction as a function of the ramp's angle of elevation. This angle where the object just begins to slide is known as the **angle of repose**.



Diagram





Newton's 2nd Law Applied to Pseudo-FBD:

$$F_{net_x} = F_{f_s} - mg\sin\theta = ma_x \xrightarrow{a_x=0} F_{f_s} = mg\sin\theta$$

$$F_{net_y} = F_N - mg\cos\theta = ma_y \xrightarrow{a_y=0} F_N = mg\cos\theta$$

Combine these equations using definition of coefficient of friction:

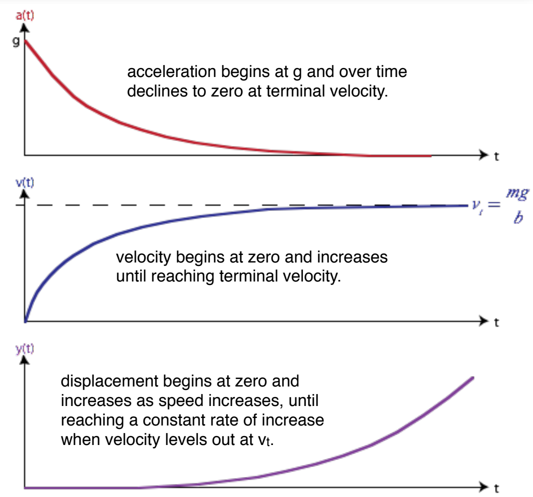
$$F_f = \mu F_N \rightarrow \mu = \frac{F_f}{F_N} = \frac{mg\sin\theta}{mg\cos\theta} \rightarrow \mu = \tan\theta$$

Drag Forces

AP-C Objectives (Drag Forces)

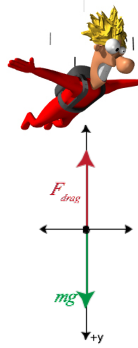
▼ 1. Students should understand the effect of drag forces on the motion of an object, so they can:

- a. Find the terminal velocity of an object moving vertically under the influence of a retarding force dependent on velocity.
- b. Describe qualitatively, with the aid of graphs, the acceleration, velocity, and displacement of such a particle when it is released from rest or is projected vertically with specified initial velocity.
- c. Use Newton's Second Law to write a differential equation for the velocity of the object as a function of time.
- d. Use the method of separation of variables to derive the equation for the velocity as a function of time from the differential equation that follows from Newton's Second Law.
- e. Derive an expression for the acceleration as a function of time for an object falling under the influence of drag forces.



Sometimes the frictional force is a function of an object's velocity (such as air resistance). These forces are called drag, or retarding, forces.

Assume we drop Matthew from an airplane. Typically, the drag forces on a free-falling object take the form $F_{drag}=bv$ or $F_{drag}=cv^2$, where b and c are constants. For the sake of this problem, let's assume $F_{drag}=bv$.



$$F_{net_y} = mg - F_{drag} = ma_y \xrightarrow{F_{drag}=bv} mg - bv = ma_y$$

Initially at time t=0, velocity=0, therefore a=g.
After a long time, however, Matthew reaches terminal velocity (v_t) and a=0.
At this point, F_{drag}=mg.

$$mg - bv = ma_y \xrightarrow[v=v_t]{a_y=0} mg - bv_t = 0 \rightarrow v_t = \frac{mg}{b}$$

To find velocity itself as a function of time, we need to go back to our Newton's 2nd Law equation. This is actually a differential equation, in which velocity and its derivative, acceleration, are both in the same equation. We can solve this by using the technique known as **separation of variables**.

$$mg - bv = ma_y \xrightarrow{a_y=\frac{dv}{dt}} mg - bv = m\frac{dv}{dt} \rightarrow \frac{mg}{b} - v = \frac{m}{b}\frac{dv}{dt} \xrightarrow{v_t=\frac{mg}{b}} v_t - v = \frac{m}{b}\frac{dv}{dt} \rightarrow$$

$$\frac{dv}{v_t - v} = \frac{b}{m}dt \rightarrow \frac{dv}{v - v_t} = -\frac{b}{m}dt$$

Now, with our velocity variables separated from our time variables, we can integrate both sides:

$$\int_0^v \frac{dv}{v-v_t} = \int_0^t -\frac{b}{m}dt \xrightarrow[\int \frac{du}{u}=\ln u + C]{\substack{u=v-v_t \\ du=dv}} \ln(v-v_t)\Big|_0^v = -\frac{b}{m}t \rightarrow$$

And the rest is algebra to solve for velocity itself:

$$\ln(v-v_t) - \ln(-v_t) = -\frac{b}{m}t \xrightarrow{\ln a - \ln b = \ln\frac{a}{b}} \ln\left(\frac{v-v_t}{-v_t}\right) = -\frac{b}{m}t \rightarrow \ln\left(\frac{v_t - v}{v_t}\right) = -\frac{b}{m}t \rightarrow$$

$$\ln\left(1-\frac{v}{v_t}\right) = -\frac{b}{m}t \rightarrow e^{\ln\left(1-\frac{v}{v_t}\right)} = e^{-\frac{b}{m}t} \rightarrow 1 - \frac{v}{v_t} = e^{-\frac{b}{m}t} \rightarrow \frac{v}{v_t} = 1 - e^{-\frac{b}{m}t} \rightarrow$$

$$\boxed{v = v_t\left(1-e^{-\frac{b}{m}t}\right) \xrightarrow{v_t=\frac{mg}{b}} v = \frac{mg}{b}\left(1-e^{-\frac{b}{m}t}\right)}$$

Now that we know v(t), we can solve for the acceleration by taking the derivative of v(t) with respect to time.

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(v_t - v_t e^{-\frac{b}{m}t}\right) = \frac{d}{dt}\left(-v_t e^{-\frac{b}{m}t}\right) = -v_t\frac{d}{dt}\left(e^{-\frac{b}{m}t}\right) \rightarrow a = -v_t e^{-\frac{b}{m}t}\left(-\frac{b}{m}\right) \rightarrow$$

$$\boxed{a = -\left(\frac{mg}{b}\right)\left(-\frac{b}{m}\right)e^{-\frac{b}{m}t} \rightarrow a = ge^{-\frac{b}{m}t}}$$

Note the form of the solution. For these types of problems, you will always see something of the form $1-e^{\frac{-t}{\tau}}$ or $e^{\frac{-t}{\tau}}$ where τ is some time constant. You can use the shape of the graph at t=0 and as t approaches infinity to tell you which form to use. It is often advantageous to guess at the form of the solution using what you know about the problem, prior to diving into a full solution.

AP-C Objectives (from College Board Learning Objectives for AP Physics)

▼ 1. Newton's 3rd Law

- a. Students should understand Newton's Third Law so that, for a given system, they can identify the force pairs and the objects on which they act, and state the magnitude and direction of each force.
- b. Students should be able to apply Newton's Third Law in analyzing the force of contact between two objects that accelerate together along a horizontal or vertical line, or between two surfaces that slide across one another.
- c. Students should know that the tension is constant in a light string that passes over a massless pulley and should be able to use this fact in analyzing the motion of a system of two objects joined by a string.
- d. Students should be able to solve problems in which application of Newton's laws leads to two or three simultaneous linear equations involving unknown forces or accelerations.

When an interaction takes place between two systems, each system exerts a force on the other, and these two forces are equal in magnitude and opposite in direction.

▼ Action-Reaction Pairs (aka Force Pairs)

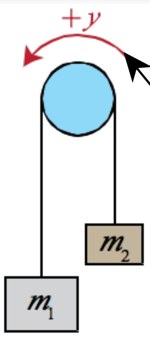
A brick rests on a table. What are the action-reaction pairs?

1) Brick attracts the Earth gravitationally / Earth attracts the brick gravitationally.

2) Brick causes table to 'sag' / Table applied normal force on brick.

AP-C Objectives (Pulleys and Atwood Machines)

1. Students should know that the tension is constant in a light string that passes over a massless pulley and should be able to use this fact in analyzing the motion of a system of two objects joined by a string.
2. Students should be able to solve problems in which application of Newton's laws leads to two or three simultaneous linear equations involving unknown forces or accelerations.

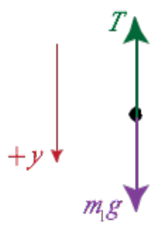


Ideal pulleys are frictionless and massless and add no inertia to the system. The tension is constant in a light string passing over an ideal pulley.

First, adopt a sign convention for positive and negative motion.

Then, analyze each mass separately using Newton's 2nd Law.

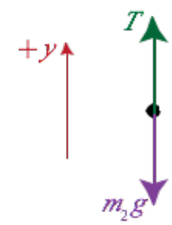
m_1 FBD



$$F_{net_y} = T - m_2 g = m_2 a \rightarrow$$

$$T = m_2 a + m_2 g$$

m_2 FBD



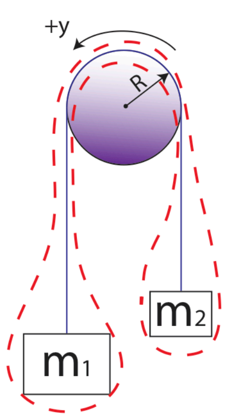
$$F_{net_y} = m_1 g - T = m_1 a \rightarrow$$

$$m_1 g - (m_2 a + m_2 g) = m_1 a \rightarrow$$

$$m_1 g - m_2 g = m_1 a + m_2 a \rightarrow g(m_1 - m_2) = a(m_1 + m_2) \rightarrow$$

$$a = g\left(\frac{m_1 - m_2}{m_1 + m_2}\right)$$

Alternately, you can analyze the system as a whole.



Drawing a dashed line around the system, you can directly write an appropriate Newton's 2nd Law equation for the entire system.

$$m_1 g - m_2 g = (m_1 + m_2) a$$

$$a = g\frac{(m_1 - m_2)}{(m_1 + m_2)}$$