AP-C Electric Potential

AP-C Objectives (from College Board Learning Objectives for AP Physics)

▼ 1. Electric potential due to point charges
- ● a. Determine the electric potential in the vicinity of one or more point charges.
- ● b. Calculate the electrical work done on a charge or use conservation of energy to determine the speed of a charge that moves through a specified potential difference.
- ● c. Determine the direction and approximate magnitude of the electric field at various positions given a sketch of equipotentials.
- ● d. Calculate the potential difference between two points in a uniform electric field, and state which point is at the higher potential.
- ● e. Calculate how much work is required to move a test charge from one location to another in the field of fixed point charges.
- ● f. Calculate the electrostatic potential energy of a system of two or more point charges, and calculate how much work is require to establish the charge system.
- ● g. Use integration to determine the electric potential difference between two points on a line, given electric field strength as a function of position on that line.
- ● h. State the relationship between field and potential, and define and apply the concept of a conservative electric field.

▼ 2. Electric potential due to other charge distributions
- ● a. Calculate the electric potential on the axis of a uniformly charged disk.
- ● b. Derive expressions for electric potential as a function of position for uniformly charged wires, parallel charged plates, coaxial cylinders, and concentric spheres.

▼ 3. Conductors
- ▼ a. Understand the nature of electric fields and electric potential in and around conductors.
  - ● i. Explain the mechanics responsible for the absence of electric field inside a conductor, and know that all excess charge must reside on the surface of the conductor.
  - ● ii. Explain why a conductor must be an equipotential, and apply this principle in analyzing what happens when conductors are connected by wires.
  - ● iii. Show that the field outside a conductor must be perpendicular to the surface.
- ● b. Graph the electric field and electric potential inside and outside a charged conducting sphere.
- ▼ c. Understand induced charge and electrostatic shielding.
  - ● i. Explain why there can be no electric field in a charge-free region completely surrounded by a single conductor.
  - ● ii. Explain why the electric field outside a closed conducting surface cannot depend on the precise location of charge in the space enclosed by the conductor.

▼ 4. Capacitors
- ▼ a. Understand the definition and function of capacitance.
  - ● i. Relate stored charge and voltage for a capacitor.
  - ● ii. Relate voltage, charge, and stored energy for a capacitor.
  - ● iii. Recognize situations in which energy stored in a capacitor is converted to other forms.
- ▼ b. Understand the physics of a parallel-plate capacitor.
  - ● i. Describe the electric field inside the capacitor and relate the strength of the field to the potential difference and separation between the plates.
  - ● ii. Relate the electric field to the charge density on the plates.
  - ● iii. Derive an expression for the capacitance of a parallel-plate capacitor.
  - ● iv. Determine how changes in the geometry of the capacitor will affect its capacitance.
  - ● v. Derive and apply expressions for the energy stored in a parallel-plate capacitor as well as the energy density in the field between the plates.
  - ● vi. Analyze situations in which capacitor plates are moved apart or closer together, or in which a conducting slab is inserted between capacitor plates.
- ● c. Describe the electric field inside cylindrical and spherical capacitors.
- ● d. Derive an expression for the capacitance of cylindrical and spherical capacitors.

▼ 5. Dielectrics
- ● a. Describe how insertion of a dielectric between the plates of a charged parallel-plate capacitor affects its capacitance and the field strength and voltage between the plates.
- ● b. Analyze situations in which a dielectric slab is inserted between the plates of a capacitor.

Electric Potential due to Point Charges

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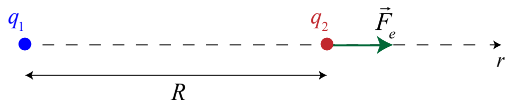
   - b. Calculate the electrical work done on a charge or use conservation of energy to determine the speed of a charge that moves through a specified potential difference.
   - c. Determine the direction and approximate magnitude of the electric field at various positions given a sketch of equipotentials.
   - d. Calculate the potential difference between two points in a uniform electric field, and state which point is at the higher potential.
   - e. Calculate how much work is required to move a test charge from one location to another in the field of fixed point charges.
   - f. Calculate the electrostatic potential energy of a system of two or more point charges, and calculate how much work is require to establish the charge system.
   - g. Use integration to determine the electric potential difference between two points on a line, given electric field strength as a function of position on that line.
   - h. State the relationship between field and potential, and define and apply the concept of a conservative electric field.

Electrical Potential Energy due to a Point Charge

Determine the work required to take a point charge q_2 from infinity (U=0) to some point a distance R away from point charge q_1.



Independent of path since F_e is a conservative force!

$$W=\int_{r=\infty}^{r=R}-\vec{F}_e\bullet d\vec{r}=\int_{r=R}^{r=\infty}\vec{F}_e\bullet d\vec{r}=\int_R^{\infty}\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}dr=\frac{q_1q_2}{4\pi\varepsilon_0}\int_R^{\infty}\frac{dr}{r^2}\rightarrow W=\frac{q_1q_2}{4\pi\varepsilon_0}\left(\frac{-1}{r}\Big|_R^{\infty}\right)\rightarrow U=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{R}$$

Electric Force from Electric Potential Energy

$$F=-\frac{dU}{dl}=-\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}\right)=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

Electric Potential due to a Point Charge

Electric potential (voltage) is the work per unit charge required to bring a charge from infinity to some point R in an electric field.

$$V=\frac{W}{q}=\frac{1}{4\pi\varepsilon_0 q}\frac{q_1q_2}{R}=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

If there are multiple charges, just add up the electric potentials due to each of the charges.

$$V=\sum_i\frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}=\frac{1}{4\pi\varepsilon_0}\sum_i\frac{q_i}{r_i}$$

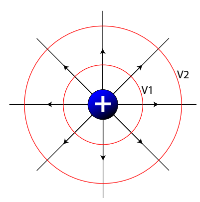
Equipotentials

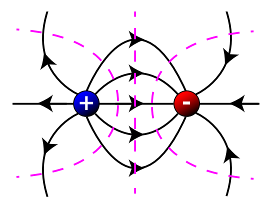
Equipotentials are surfaces with constant potential, similar to altitude lines on a topographic map.

Equipotential lines always run perpendicular to electric field lines.

The work done in moving a particle through space is zero if its path begins and ends anywhere on the same equipotential line since the electric force is conservative.

Electric field points from high potential to low potential.





Finding Electric Field from Electric Potential

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\rightarrow\frac{dV}{dr}=\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right)=\frac{q}{4\pi\varepsilon_0}\frac{d}{dr}\left(\frac{1}{r}\right)=\frac{-q}{4\pi\varepsilon_0}\frac{d}{dr}\left(r^{-1}\right)\rightarrow$$

$$\frac{dV}{dr}=\frac{-q}{4\pi\varepsilon_0 r^2}\rightarrow\frac{dV}{dr}\hat{r}=\frac{-q}{4\pi\varepsilon_0 r^2}\hat{r}=-\vec{E}\rightarrow\vec{E}=-\frac{dV}{dr}\hat{r}$$

Finding Electric Potential from Electric Field

$$V=\frac{W}{q}=\frac{1}{q}\int_r^{\infty}\vec{F}_e\bullet d\vec{r}=\int_r^{\infty}\frac{\vec{F}_e}{q}\bullet d\vec{r}\xrightarrow{\vec{E}=\frac{\vec{F}_e}{q}}V=\int_r^{\infty}\vec{E}\bullet d\vec{r}$$

$$\Delta V=V_B-V_A=-\int_A^B\vec{E}\bullet d\vec{l}=\frac{\Delta U}{q}$$

Sample Problem: Finding Electric Potential due to a Collection of Point Charges

Find the electric potential at the origin due to the following charges: +2μC at (3,0); –5μC at (0,5); and +1μC at (4,4).

$$V=\frac{1}{4\pi\varepsilon_0}\sum_i\frac{q_i}{r_i}=\frac{1}{4\pi\varepsilon_0}\left(\frac{+2\times10^{-6}}{3}+\frac{-5\times10^{-6}}{5}+\frac{+1\times10^{-6}}{\sqrt{4^2+4^2}}\right)=-1410V$$

Sample Problem: Electric Field from Potential

Given an electric potential of $V(x)=5x^2-7x$, find the magnitude and direction of the electric field at x=3 m.

$$\vec{E}=-\frac{dV}{dr}\hat{r}=-\frac{d}{dx}(5x^2-7x)\hat{i}=-(10x-7)\hat{i}=(7-10x)\hat{i}\xrightarrow{x=3m}$$

$$\vec{E}=(7-10(3))\hat{i}=-23\,{}^{V}\!/_{m}\,\hat{i}$$

Sample Problem: Speed of an Electron Released in an E Field

An electron is released from rest in a uniform electric field of 500 N/C. What is its velocity after it has traveled one meter?

$$\Delta K=-\Delta U\xrightarrow{\Delta U=-q\int_A^B\vec{E}\bullet d\vec{r}}\Delta K=q\int_A^B\vec{E}\bullet d\vec{r}\rightarrow$$

$$\Delta K=qE\int_A^B dr=qE\Delta r\rightarrow\tfrac{1}{2}mv^2=qE\Delta r\rightarrow$$

$$v=\sqrt{\frac{2qE\Delta r}{m}}=\sqrt{\frac{2(1.6\times10^{-19})(500)(1)}{9.11\times10^{-31}}}\rightarrow$$

$$v=1.33\times10^7\,{}^{m}\!/_{s}$$

Work Required to Establish a System of Point Charges

Two point charges (5 μC and 2 μC) are placed 0.5 meters apart. How much work was required to establish the charge system? What is the electric potential halfway between the two charges?

$$U_e=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}=\frac{1}{4\pi\varepsilon_0}\frac{(5\times10^{-6})(2\times10^{-6})}{0.5}=0.18J$$

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}+\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r}=\frac{1}{4\pi\varepsilon_0}\frac{5\times10^{-6}}{0.25}+\frac{1}{4\pi\varepsilon_0}\frac{2\times10^{-6}}{0.25}=252kV$$

Electric Potential due to Other Charge Distributions

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▼ 1. Electric potential due to other charge distributions

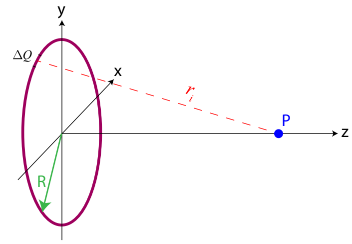
- a. Calculate the electric potential on the axis of a uniformly charged disk.
- b. Derive expressions for electric potential as a function of position for uniformly charged wires, parallel charged plates, coaxial cylinders, and concentric spheres.

▼ Electric Potential on the Axis of a Uniformly Charged Ring

Find the electric potential on the axis of a uniformly charged ring of radius R and total charge Q at point P located a distance z from the center of the ring.

$$V_P = \sum_i V_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{Q_i}{r_i} \to V_P = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} \xrightarrow{r=r_i}$$

$$V_P = \frac{1}{4\pi\varepsilon_0 r_i}\int dq \xrightarrow[\int dq=Q]{r_i=\sqrt{z^2+R^2}} V_P = \frac{Q}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2+R^2}}$$



▼ Electric Potential on the Axis of a Uniformly Charged Disk

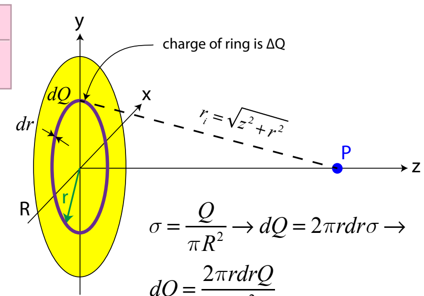
Find the electric potential on the axis of a uniformly charged disk of radius R and total charge Q at point P located a distance z from the center of the disk.

$$V_P = \sum_i V_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{dQ}{r_i} \to V_P = \frac{1}{4\pi\varepsilon_0}\int_{r=0}^{R} \frac{2\pi r dr Q}{\pi R^2 r_i} \to$$

$$V_P = \frac{2Q}{4\pi\varepsilon_0 R^2}\int_0^R \frac{r dr}{r_i} \to V_P = \frac{Q}{2\pi\varepsilon_0 R^2}\int_0^R (z^2+r^2)^{-\frac{1}{2}} r\, dr \to$$

$$V_P = \frac{Q}{2\pi\varepsilon_0 R^2}\frac{1}{2}\int_0^R (z^2+r^2)^{-\frac{1}{2}} 2r\, dr \xrightarrow[du=2rdr]{u=z^2+r^2}$$

$$V_P = \frac{Q}{4\pi\varepsilon_0 R^2}\left[2(z^2+r^2)^{\frac{1}{2}}\right]_0^R = \frac{Q}{2\pi\varepsilon_0 R^2}(\sqrt{z^2+r^2}-z)$$

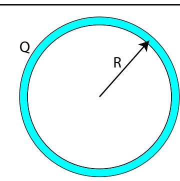


$$\sigma = \frac{Q}{\pi R^2} \to dQ = 2\pi r dr \sigma \to$$

$$dQ = \frac{2\pi r dr Q}{\pi R^2}$$

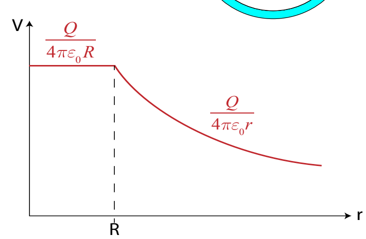
▼ Electric Potential due to a Spherical Shell of Charge

Find the electric potential both inside and outside a uniformly charged shell of radius R and total charge Q.



$$V_{outside} = -\int \vec{E}\bullet d\vec{l} = -\int_{r=\infty}^{r} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dr = \frac{-Q}{4\pi\varepsilon_0}\int_{r=\infty}^{r} r^{-2}\,dr = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{-1}{r}\right)_{\infty}^{r} \to$$

$$V_{outside} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{\infty}\right) \to V_{outside} = \frac{Q}{4\pi\varepsilon_0 r}$$



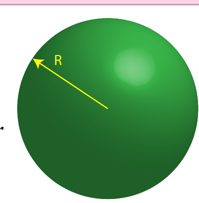
$$V_{inside} = -\int \vec{E}\bullet d\vec{l} = -\int_{\infty}^{R} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dr - \int_R^r 0\,dr = \frac{Q}{4\pi\varepsilon_0 R}$$

▼ Electric Potential Inside a Uniformly Charged Solid Insulating Sphere

Find the electric field and electric potential inside a uniformly charged solid insulating sphere of radius R and total charge Q.

▼ Strategy

First find the electric field. Choose a sphere as our Gaussian surface.



$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3} \to$$

$$\rho = \frac{3Q}{4\pi R^3}$$

$$\oint \vec{E}\bullet d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \to E(4\pi r^2) = \frac{\rho V}{\varepsilon_0} = \frac{3Q}{4\pi\varepsilon_0 R^3}\frac{4\pi r^3}{3} \to E = \frac{Q}{4\pi\varepsilon_0 R^3}r$$

▼ Strategy

Next, integrate to find the electric potential. Note that your total integration from infinity to r must be done piece-wise since the electric field is discontinuous.

$$V = -\int_{\infty}^{r} \vec{E}\bullet d\vec{r} \xrightarrow{piece-wise} V = -\int_{\infty}^{R} \vec{E}\bullet d\vec{r} - \int_R^r \vec{E}\bullet d\vec{r} = -\int_{\infty}^{R} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dr - \int_R^r \frac{1}{4\pi\varepsilon_0 R^3} r\,dr \to$$

$$V = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 R^3}\int_R^r r\,dr = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 R^3}\left(\frac{r^2}{2}-\frac{R^2}{2}\right) = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Qr^2}{8\pi\varepsilon_0 R^3} + \frac{QR^2}{8\pi\varepsilon_0 R^3} \to$$

$$V = \frac{3Q}{8\pi\varepsilon_0 R} - \frac{Qr^2}{8\pi\varepsilon_0 R^3} \to V = \frac{Q}{8\pi\varepsilon_0 R}\left(3-\frac{r^2}{R^2}\right)$$

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- 1. Conductors
  - a. Understand the nature of electric fields and electric potential in and around conductors.
    - i. Explain the mechanics responsible for the absence of electric field inside a conductor, and know that all excess charge must reside on the surface of the conductor.
    - ii. Explain why a conductor must be an equipotential, and apply this principle in analyzing what happens when conductors are connected by wires.
    - iii. Show that the field outside a conductor must be perpendicular to the surface.
  - b. Graph the electric field and electric potential inside and outside a charged conducting sphere.
  - c. Understand induced charge and electrostatic shielding.
    - i. Explain why there can be no electric field in a charge-free region completely surrounded by a single conductor.
    - ii. Explain why the electric field outside a closed conducting surface cannot depend on the precise location of charge in the space enclosed by the conductor.

Charges in a Conductor

Charges are free to move in conductors. At electrostatic equilibrium, there are no moving charges in a conductor, therefore there is no net force, and the electric field inside the conductor must be zero. Gauss's Law therefore states that the charge enclosed must be zero. All excess charge on a conductor lies on the surface of the conductor, and the field on the surface of the conductor must be perpendicular to the surface, otherwise the charges would move.

Electric Field at the Surface of a Conductor

Looking at just the outer surface of a conductor:

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \xrightarrow[Q=\sigma A]{Symmetry\ exists} EA = \frac{\sigma A}{\varepsilon_0} \rightarrow E = \frac{\sigma}{\varepsilon_0}$$

This should make sense… you have the largest electric field where you have the highest surface charge density.

Charge Distribution in a Hollow Conductor

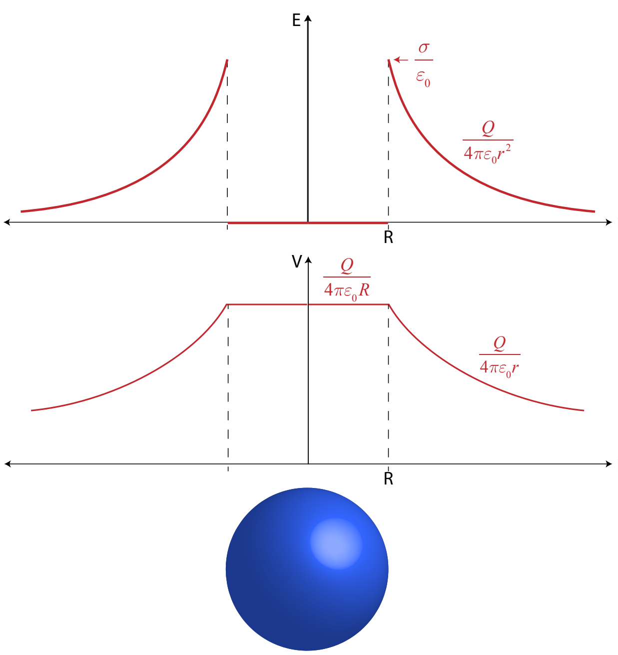
In a hollow conductor, you can determine the location of charge by utilizing Gauss's Law. Choose a Gaussian surface in the metal of the hollow conductor, making note that the electric field inside the conductor must be zero.

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \xrightarrow{\vec{E}=0} Q = 0$$

Therefore, the charge must remain on the outer surface. The entire conductor is at equipotential, and field lines must run perpendicular to the conducting surface.

Electric Field and Potential Due to a Conducting Sphere

Graph the electric field and the electric potential both inside and outside a solid conducting sphere.



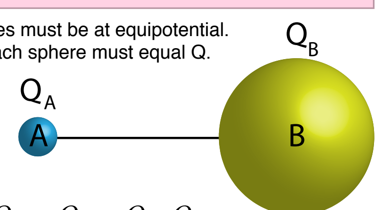
Faraday Cage

Any hollow conductor has zero electric field in its interior. This allows for hollow conductors to be utilized to isolate regions completely from electric fields. In this configuration, a hollow conductor is known as a Faraday Cage.

Sample Problem: Conducting Spheres Connected by a Wire

Two conducting spheres, A and B, are placed a large distance from each other. The radius of Sphere A is 5 cm, and the radius of Sphere B is 20 cm. A charge Q of 200 nC is placed on Sphere A, while Sphere B is uncharged. The spheres are then connected by a wire. Calculate the charge on each sphere after the wire is connected.

Once connected by a wire, the spheres must be at equipotential. Further, the sum of the charges on each sphere must equal Q.



$$Q = Q_A + Q_B \rightarrow$$
$$Q_B = Q - Q_A$$

$$V_A = \frac{Q_A}{4\pi\varepsilon_0 R_A} = \frac{Q_B}{4\pi\varepsilon_0 R_B} \rightarrow \frac{Q_A}{R_A} = \frac{Q_B}{R_B} = \frac{Q - Q_A}{R_B} \rightarrow$$

$$Q_A R_B = QR_A - Q_A R_A \rightarrow Q_A R_B + Q_A R_A = QR_A \rightarrow$$

$$Q_A = \frac{QR_A}{R_A + R_B} = \frac{(200nC)(5cm)}{(5cm + 20cm)} = 40nC$$

$$Q_B = 200nC - 40nC = 160nC$$

Capacitors

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   a. Understand the definition and function of capacitance.
      i. Relate stored charge, voltage, and stored energy for a capacitor.
   b. Understand the physics of a parallel-plate capacitor.
      i. Describe the electric field inside the capacitor and relate the strength of the field to the potential difference and separation between the plates.
      ii. Relate the electric field to the charge density on the plates.
      iii. Derive an expression for the capacitance of a parallel-plate capacitor.
      iv. Determine how changes in the geometry of the capacitor will affect its capacitance.
   c. Describe the electric field inside cylindrical and spherical capacitors.
   d. Derive an expression for the capacitance of cylindrical and spherical capacitors.

What is a Capacitor?

A capacitor is a device which stores electrical energy. Consisting of two conducting plates separated by an insulator, a capacitor holds opposite charges on each plate with a potential difference across the plates.

Capacitance

Capacitance is the ratio of the charge separated on the plates of the capacitor to the potential difference between the plates. Units of capacitance are coulombs/volt, or farads.

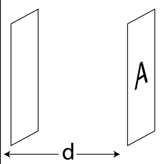
$$C=\frac{Q}{V}$$

Calculating Capacitance

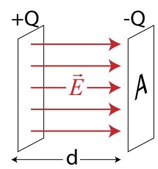
1. Assume a charge of +Q and -Q on each of the conductors.
2. Find the electric field between the conductors.
3. Calculate V by integrating the electric field. $V=-\int \vec{E}\bullet d\vec{l}$
4. Utilize C=Q/V to solve for capacitance.

Capacitance of Parallel Plates

Determine the capacitance of two identical parallel plates of area A separated by a distance d.



1. Assume a charge of +Q and -Q on each plate.
2. Electric field due to a plane of charge is σ/ε_0.
3. $V=-\int \vec{E}\bullet d\vec{l}=Ed\xrightarrow{E=\sigma/\varepsilon_0}V=\frac{\sigma d}{\varepsilon_0}\xrightarrow{\sigma=Q/A}V=\frac{Qd}{\varepsilon_0 A}$
4. $C=\frac{Q}{V}=\frac{Q}{Qd/\varepsilon_0 A}=\frac{\varepsilon_0 A}{d}$



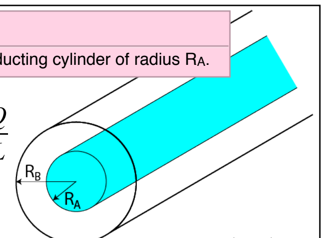
Electric Field Between Parallel Plates

Note the electric field between the plates: E=V/d is a constant as long as you stay far from the edges of the plates.

Capacitance of a Cylindrical Capacitor

Determine the capacitance of a long, thin hollow conducting cylinder of radius R_B surrounding a long solid conducting cylinder of radius R_A.

$$\lambda=\frac{Q}{L}$$



1. Assume a charge of +Q and -Q on each of the cylinders.
2. Determine the electric field between the cylinders.

$$\oint \vec{E}\bullet d\vec{A}=\frac{Q_{enc}}{\varepsilon_0}\xrightarrow{\lambda=\frac{Q}{L}}E(2\pi rL)=\frac{\lambda L}{\varepsilon_0}\rightarrow E=\frac{\lambda}{2\pi\varepsilon_0 r}$$

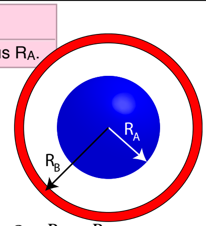
3. Calculate V by integrating the electric field.

$$V=-\int \vec{E}\bullet d\vec{l}=-\int_{r=R_A}^{R_B}\frac{\lambda}{2\pi\varepsilon_0 r}dr=-\frac{\lambda}{2\pi\varepsilon_0}\int_{R_A}^{R_B}\frac{dr}{r}=-\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R_B}{R_A}\right)=\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R_A}{R_B}\right)\xrightarrow{\lambda=Q/L}V=\frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{R_A}{R_B}\right)$$

4. Find C using C=Q/V: $C=\frac{Q}{V}=\frac{Q}{\frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{R_A}{R_B}\right)}=\frac{2\pi\varepsilon_0 L}{\ln(R_A/R_B)}$

Capacitance of a Spherical Capacitor

Determine the capacitance of a thin hollow conducting shell of radius R_B concentric around a solid conducting sphere of radius R_A.



1. Assume a charge of +Q and -Q on each of the cylinders.
2. Determine the electric field between the shells.

$$\int \vec{E}\bullet d\vec{A}=\frac{Q_{enc}}{\varepsilon_0}\rightarrow E(4\pi r^2)=\frac{Q}{\varepsilon_0}\rightarrow E=\frac{Q}{4\pi\varepsilon_0 r^2}$$

3. Calculate V by integrating the electric field.

$$V=-\int \vec{E}\bullet d\vec{l}=-\int_{R_A}^{R_B}\frac{Q}{4\pi\varepsilon_0 r^2}dr=\frac{-Q}{4\pi\varepsilon_0}\int_{R_A}^{R_B}r^{-2}dr=\frac{-Q}{4\pi\varepsilon_0}\left(\frac{-1}{r}\right)\Bigg|_{R_A}^{R_B}=\frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{R_A}-\frac{1}{R_B}\right)=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_B}-\frac{1}{R_A}\right)=\frac{Q}{4\pi\varepsilon_0}\frac{R_A-R_B}{R_A R_B}\xrightarrow{R_A<R_B}$$

$$|V|=\frac{Q}{4\pi\varepsilon_0}\frac{R_B-R_A}{R_A R_B}$$

4. Find C using C=Q/V: $C=\frac{Q}{V}=\frac{Q}{\frac{Q}{4\pi\varepsilon_0}\frac{R_B-R_A}{R_A R_B}}=\frac{4\pi\varepsilon_0 R_A R_B}{R_B-R_A}$

AP-C Objectives (from College Board Learning Objectives for AP Physics)

- 1. Capacitors
  - a. Understand the definition and function of capacitance.
    - i. Relate stored charge, voltage, and stored energy for a capacitor.
    - ii. Recognize situations in which energy stored in a capacitor is converted to other forms.
  - b. Understand the physics of a parallel-plate capacitor.
    - i. Derive and apply expressions for the energy stored in a parallel-plate capacitor as well as the energy density in the field between the plates.
    - ii. Analyze situations in which capacitor plates are moved apart or closer together, or in which a conducting slab is inserted between capacitor plates.
  - c. Describe how insertion of a dielectric between the plates of a charged parallel-plate capacitor affects its capacitance and the field strength and voltage between the plates.

Energy Stored in a Capacitor

Work is done in charging a capacitor, allowing the capacitor to store energy. If you consider two uncharged conductors in close proximity, the potential difference in moving some amount of charge q from the negative to the positive plate is q/C. Moving more charge increases the potential, therefore the electric potential energy of the charge and the capacitor must also increase.

$$U_{cap} = \int_{q=0}^{Q} \frac{q}{C}\,dq = \frac{1}{C}\int_{0}^{Q} q\,dq = \frac{1}{C}\frac{Q^2}{2} = \frac{1}{2}\frac{Q^2}{C} \xrightarrow{C=Q/V} U = \frac{1}{2}QV \xrightarrow[Q=CV]{C=Q/V} U = \frac{1}{2}CV^2$$

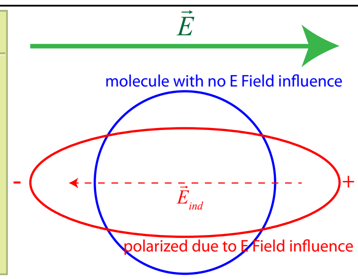
Field Energy Density

Consider a parallel plate capacitor of plate area A and plate separation d. We can think of the energy stored in the capacitor as the work required to create the electric field between the plates. Therefore, a capacitor stores energy by creating an electric field. The amount of energy stored as electric field per unit volume between the plates is known as the field energy density u_e.

$$U = \frac{1}{2}CV^2 \xrightarrow[E=V/d \to V=Ed]{C=\varepsilon_0 A/d} U = \frac{1}{2}\frac{\varepsilon_0 A}{d}E^2 d^2 = \frac{1}{2}\varepsilon_0 E^2 Ad \xrightarrow{Ad=Volume} \frac{U}{Volume} = u_e = \frac{1}{2}\varepsilon_0 E^2$$

Dielectrics

Dielectrics are insulating materials which are placed between the plates of a capacitor to increase the device's capacitance. When a dielectric is placed between the plates of a capacitor, the electric field between the plates is weakened. This is due to the molecules of the dielectric becoming polarized in the electric field created by the potential difference of the capacitor plates, creating an opposite electric field. The greater the amount of polarization, the greater the reduction in the electric field. Therefore, for a fixed charge charge on the plates Q, the voltage decreases, increasing the capacitance (C=Q/V).



Dielectric Constant (κ)

The amount by which the capacitance is increased when a dielectric is introduced between the plates of a capacitor is known as the dielectric constant (**κ**) of the dielectric. This constant also corresponds to the amount the electric field strength is reduced due to introduction of the dielectric. The more the molecules / atoms of the dielectric are polarized, the greater the dielectric constant.

$$C = \frac{\kappa\varepsilon_0 A}{d} \xrightarrow{\varepsilon=\kappa\varepsilon_0} C = \frac{\varepsilon A}{d}$$

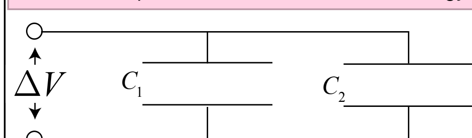
$$\vec{E}_{net} = \frac{\vec{E}_{w/o\ dielectric}}{\kappa}$$

$$\varepsilon = \kappa\varepsilon_0$$

permittivity of the dielectric

Capacitors in Parallel

Capacitors in parallel have the same voltage across their plates due to conservation of energy.



$$C_{eq} = \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2 \rightarrow$$

$$C_{eq} = C_1 + C_2 + \ldots$$

Capacitors in Series

Capacitors in series have the same charge because each plate must obtain charge from the next plate due to conservation of charge.

$$C_{eq} = \frac{Q}{V} = \frac{Q}{V_1 + V_2} = \frac{Q}{\frac{Q}{C_1} + \frac{Q}{C_2}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \rightarrow$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$