AP-C Objectives (from College Board Learning Objectives for AP Physics)

- 1. Electromagnetic Induction (including Faraday's Law and Lenz's Law)
  - a. Magnetic Flux
    - i. Calculate the flux of a uniform magnetic field through a loop of arbitrary orientation.
    - ii. Use integration to calculate the flux of a non-uniform magnetic field, whose magnitude is a function of one coordinate, through a rectangular loop perpendicular to the field.
  - b. Faraday's Law and Lenz's Law
    - i. Recognize situations in which changing flux through a loop will cause an induced emf or current in the loop.
    - ii. Calculate the magnitude and direction of the induced emf and current in a loop of wire or a conducting bar when the magnitude of a related quantity such as magnetic field or area of the loop is specified as a non-linear function of time.
    - iii. Analyze the forces that act on induced currents to determine the mechanical consequences of those forces.
- 2. Inductance (including LR and LC circuits)
  - a. Concept of Inductance
    - i. Calculate the magnitude and sense of the emf in an inductor through which a specified changing current is flowing.
    - ii. Derive and apply the expression for the self-inductance of a long solenoid.
  - b. Transient and steady state behavior of DC circuits containing resistors and inductors
    - i. Apply Kirchhoff's rules to a simple LR series circuit to obtain a differential equation for the current as a function of time.
    - ii. Solve the differential equation obtained in (i) for the current as a function of time through the battery, using separation of variables.
    - iii. Calculate the initial transient currents and final steady state currents through any part of a simple series and parallel circuit containing an inductor and one or more resistors.
    - iv. Sketch graphs of the current through or voltage across the resistors or inductor in a simple series and parallel circuit.
    - v. Calculate the rate of change of current in the inductor as a function of time.
    - vi. Calculate the energy stored in an inductor that has a steady current flowing through it.
- 3. Students should be familiar with Maxwell's equations so they can associate each equation with its implications.

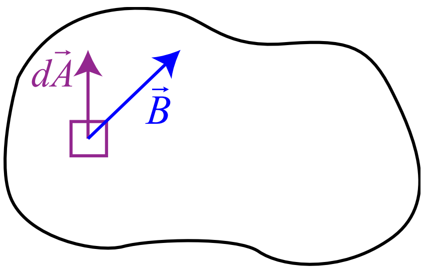
Objectives

1. Calculate the flux of a uniform magnetic field through a loop of arbitrary orientation.
2. Use integration to calculate the flux of a non-uniform magnetic field, whose magnitude is a function of one coordinate, through a rectangular loop perpendicular to the field.

▼ Units of Magnetic Flux

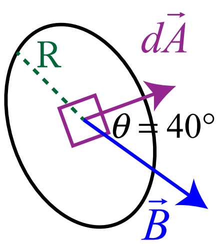
Units of magnetic flux are webers (Wb)

1 weber = 1 Tesla•m^2



$$\Phi_B = \int_{\substack{open\\surface}} \vec{B} \bullet d\vec{A}$$

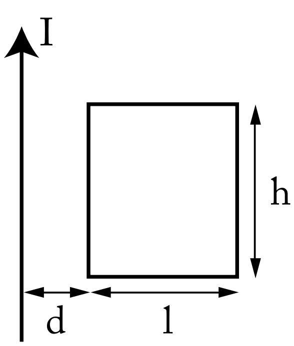
Example: Calculate the flux of a 3-Tesla uniform magnetic field through the circular loop of radius 0.2 meters with three turns of wire.



$$\Phi_B = \int_{\substack{open\\surface}} \vec{B} \bullet d\vec{A} = BA\cos\theta = B(\pi R^2)\cos\theta \rightarrow$$

$$\Phi_B = 3(3T)(\pi \times (0.2m)^2)\cos(40°) = 0.866Wb$$

Example: A long straight wire carries a current I as shown. Calculate the magnetic flux through the loop.



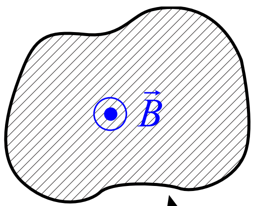
$$\Phi_B = \int_{\substack{open\\surface}} \vec{B} \bullet d\vec{A} = \int_{r=d}^{r=d+l} \frac{\mu_0 I}{2\pi r} h\,dr = \frac{\mu_0 Ih}{2\pi}\int_{d}^{d+l} \frac{dr}{r} \rightarrow$$

$$\Phi_B = \frac{\mu_0 Ih}{2\pi}\left[\ln(d+l) - \ln(d)\right] = \frac{\mu_0 Ih}{2\pi}\ln\left(\frac{d+l}{d}\right)$$

Electromagnetic Induction

Faraday's Law and Lenz's Law

1. Recognize situations in which changing flux through a loop will cause an induced emf or current in the loop.
2. Calculate the magnitude and direction of the induced emf and current in a loop of wire or a conducting bar when the magnitude of a related quantity such as magnetic field or area of the loop is specified as a non-linear function of time.
3. Analyze the forces that act on induced currents to determine the mechanical consequences of those forces.

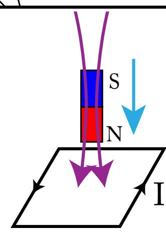


Faraday's Law: The induced emf due to a changing magnetic field is equal in magnitude to the rate of change of the magnetic flux through a surface bounded by the circuit. The direction of the induced current is given by Lenz's Law.

$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\int_{\substack{open\\surface}} \vec{B}\bullet d\vec{A} = \oint_{\substack{closed\\loop}} \vec{E}\bullet d\vec{l}$$

▼ Applying Lenz's Law

If B is increasing, you induce a clockwise current.
If B is decreasing, you induce a CCW current.



Lenz's Law: The current induced by a changing magnetic flux creates a magnetic field opposing the change in flux.

Maxwell's Equations:

Gauss's Law $\oint \vec{E}\bullet d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

Gauss's Law for Magnetism $\oint \vec{B}\bullet d\vec{A} = 0$

Faraday's Law $\oint_{\substack{closed\\loop}} \vec{E}\bullet d\vec{l} = -\frac{d}{dt}\int_{\substack{open\\surface}} \vec{B}\bullet d\vec{A}$

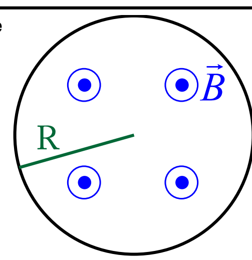
Ampere's Law* $\oint_{\substack{closed\\loop}} \vec{B}\bullet d\vec{l} = \mu_0 I$

*must modify slightly later

▼ Kirchhoff's Voltage Law Revisited

Kirchhoff's Voltage Law only holds when the magnetic flux is constant. If you have a changing magnetic flux, you must use Faraday's Law (i.e. KVL is a special case of Faraday's Law for constant magnetic flux).

Example: A magnetic field of strength B(t)=3t²-2t+1 is directed out of the plane of a circular loop of wire as shown.
A) Find the generated emf as a function of time.



$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\int_{\substack{open\\surface}} \vec{B}\bullet d\vec{A} = -\frac{d}{dt}(AB) \rightarrow$$

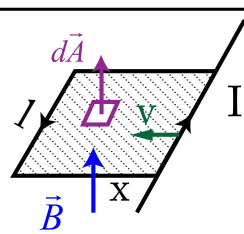
$$\varepsilon = -A\frac{dB}{dt} = -A\frac{d}{dt}(3t^2-2t+1) = -\pi R^2(6t-2)$$

B) Determine the current through the 100-ohm lamp as a function of time.

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon}{100\Omega} = \frac{-\pi R^2(6t-2)}{100\Omega}$$

C) What is the direction of the current through the loop at time t=5s?
clockwise using Lenz's Law

Example: Consider a circuit in which a current-carrying rod on rails is moved to the left with constant velocity v. If the circuit is perpendicular to a constant magnetic field, determine the induced emf in the circuit.



$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\int_{\substack{open\\surface}} \vec{B}\bullet d\vec{A} = -Bl\frac{dx}{dt} \xrightarrow{v=\frac{dx}{dt}}$$

$$\varepsilon = -Blv$$

Inductance (including LR and LC circuits)

- 1. Concept of Inductance
  - a. Calculate the magnitude and sense of the emf in an inductor through which a specified changing current is flowing.
  - b. Derive and apply the expression for the self-inductance of a long solenoid.
- 2. Transient and steady state behavior of DC circuits containing resistors and inductors
  - a. Apply Kirchhoff's rules to a simple LR series circuit to obtain a differential equation for the current as a function of time.
  - b. Solve the differential equation obtained in (a) for the current as a function of time through the battery, using separation of variables.
  - c. Calculate the initial transient currents and final steady state currents through any part of a simple series and parallel circuit containing an inductor and one or more resistors.
  - d. Sketch graphs of the current through or voltage across the resistors or inductor in a simple series and parallel circuit.
  - e. Calculate the rate of change of current in the inductor as a function of time.
  - f. Calculate the energy stored in an inductor that has a steady current flowing through it.

Self Inductance (L)

Self Inductance is the ability of a circuit to oppose the magnetic flux that is produced by the circuit itself.

Running a changing current through a circuit creates a changing magnetic field, which creates an induced emf that fights the change.

Units are henrys (H) — 1H=1V•s/A

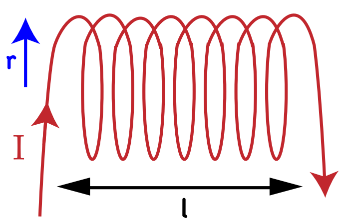
$$L = \frac{\phi_B}{I}$$

$$\varepsilon = -L\frac{dI}{dt}$$

Energy Stored in an Inductor

$$U_L = \tfrac{1}{2}LI^2$$

Example: Calculate the self-inductance of a solenoid of radius r and length L with N windings.



$$B_{inside} = \tfrac{N}{l}\mu_0 I$$ ← *See: Ampere's Law*

$$\phi_B = NB\pi r^2$$

$$L = \frac{\phi_B}{I} = \frac{NB\pi r^2}{I} = \frac{N\frac{N}{l}\mu_0 I\pi r^2}{I} = \frac{N^2}{l}\mu_0\pi r^2$$

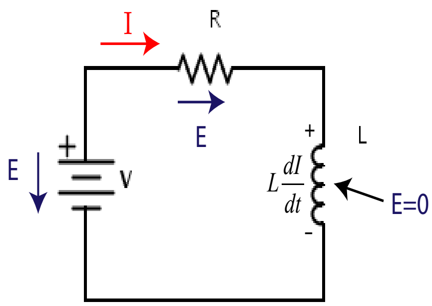
Time Constants

RC Circuit: $\tau = RC$

RL Circuit: $\tau = \frac{L}{R}$

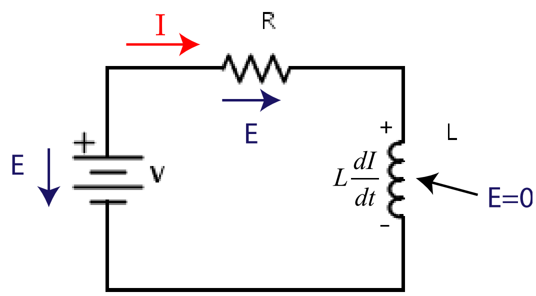
LC Circuit: $\omega = \frac{1}{\sqrt{LC}}$

RL Circuits



Inductors in Circuits

1. When circuit first turned on, inductor opposes current flow and acts like an open circuit: I(0)=0.
2. After a time, inductor keeps current going and acts as a short: I(t)=V/R.
3. After a long time, if battery is removed from circuit, inductor acts as emf source to keep current going: I(t)=V/R.
4. As the resistor dissipates power, current will decay exponentially to zero.



Apply Faraday's Law in order to find I(t).
(Can't use KVL since magnetic flux is changing)

Make a loop starting at current, showing E•dl for each component.

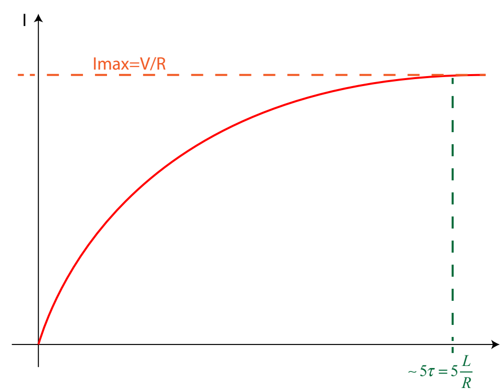
$$\oint \vec{E} \bullet d\vec{l} = -\frac{d\phi_B}{dt} = -L\frac{dI}{dt} \to IR - V = -L\frac{dI}{dt} \to$$

$$I - \frac{V}{R} = -\frac{L}{R}\frac{dI}{dt} \to \frac{dI}{I - V/R} = -\frac{R}{L}dt \to \int_{I=0}^{I} \frac{dI}{I - V/R} = \int_{t=0}^{t} -\frac{R}{L}dt \to$$

$$\ln\left(I - \frac{V}{R}\right)\Bigg|_0^I = -\frac{R}{L}t \to \ln\left(I - \frac{V}{R}\right) - \ln\left(-\frac{V}{R}\right) = -\frac{R}{L}t \to$$

$$\ln\left(\frac{I - V/R}{-V/R}\right) = -\frac{R}{L}t \to \frac{I - V/R}{-V/R} = e^{-\frac{R}{L}t} \to I - \frac{V}{R} = -\frac{V}{R}e^{-\frac{R}{L}t} \to$$

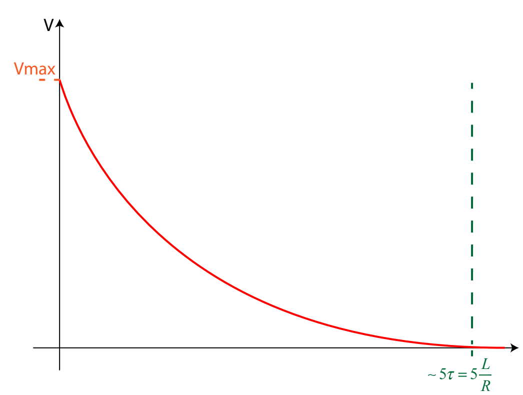
$$I = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t} \to I = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$



Differentiate to find the voltage across the inductor:

$$V_L(t) = L\frac{dI}{dt} = L\frac{d}{dt}\left(\frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right) = L\frac{V}{R}\frac{d}{dt}\left(1 - e^{-\frac{R}{L}t}\right) \to$$

$$V_L(t) = L\frac{V}{R}\left(-e^{-\frac{R}{L}t}\right)\left(-\frac{R}{L}\right) \to V_L(t) = Ve^{-\frac{R}{L}t}$$



Differentiate to find rate of change of current as a function of time:

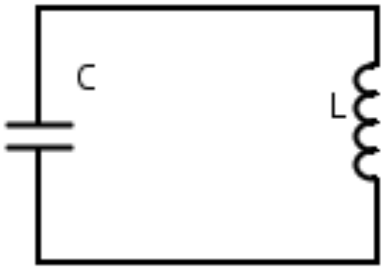
$$\frac{dI}{dt} = \frac{d}{dt}\left(\frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right) = \frac{V}{R}\frac{d}{dt}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{V}{R}\left(-e^{-\frac{R}{L}t}\right)\left(-\frac{R}{L}\right) \to$$

$$\frac{dI}{dt} = \frac{V}{L}e^{-\frac{R}{L}t}$$

Time constant for an RL circuit

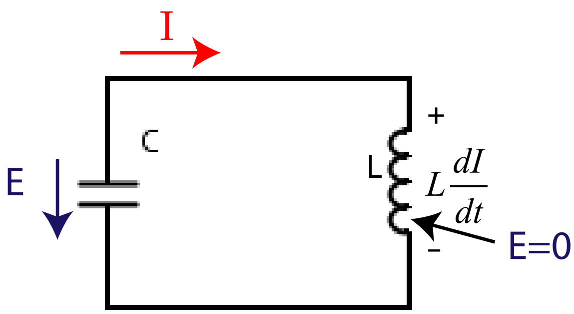
$$\tau = \frac{L}{R}$$

LC Circuits



Apply Faraday's Law in order to find I(t).
(Can't use KVL since magnetic flux is changing)

Make a loop starting at current, showing E•dl for each component.



$$\oint \vec{E} \bullet d\vec{l} = -\frac{d\phi_B}{dt} = -L\frac{dI}{dt} \rightarrow -\frac{Q}{C} = -L\frac{dI}{dt} \rightarrow$$

$$\frac{Q}{C} - L\frac{dI}{dt} = 0 \xrightarrow[\frac{dI}{dt}=-\frac{d^2Q}{dt^2}]{I=-\frac{dq}{dt}} \frac{Q}{C} - L\left(-\frac{d^2Q}{dt^2}\right) = 0 \rightarrow$$

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0 \xrightarrow{\omega=\frac{1}{\sqrt{LC}}} Q(t) = A\cos(\omega t) + B\sin(\omega t)$$

Utilize boundary conditions to find A and B

$$Q(t=0) = Q_0 \rightarrow A = Q_0$$

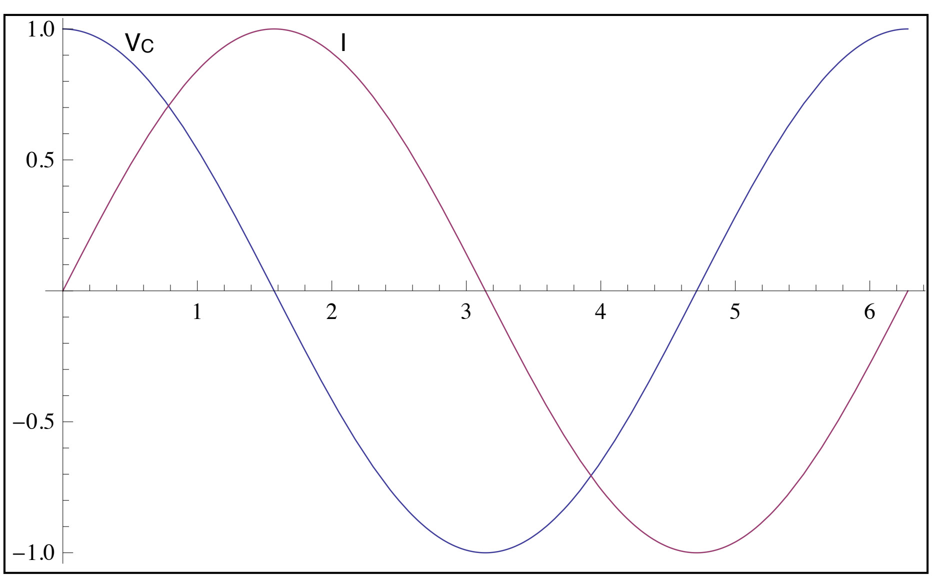
$$I(t=0) = 0 \rightarrow B = 0$$

Substitute back in to original equation to find Q, V, and I:

$$Q(t) = Q_0\cos(\omega t)$$

$$V_C(t) = \frac{Q}{C} = \frac{Q_0}{C}\cos(\omega t)$$

$$I(t) = -\frac{dQ}{dt} = \omega Q_0\sin(\omega t) = \frac{Q_0}{\sqrt{LC}}\sin(\omega t)$$



$$\omega = \frac{1}{\sqrt{LC}}$$

Students should be familiar with Maxwell's equations so they can associate each equation with its implications.

Maxwell's Equations:

Gauss's Law $$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Gauss's Law for Magnetism $$\oint \vec{B} \bullet d\vec{A} = 0$$

Faraday's Law $$\oint_{\substack{\text{closed}\\ \text{loop}}} \vec{E} \bullet d\vec{l} = -\frac{d}{dt} \int_{\substack{\text{open}\\ \text{surface}}} \vec{B} \bullet d\vec{A}$$

Ampere's Law* $$\oint_{\substack{\text{closed}\\ \text{loop}}} \vec{B} \bullet d\vec{l} = \mu_0 I$$

*must modify slightly later

Kirchhoff's Voltage Law Revisited

Kirchhoff's Voltage Law only holds when the magnetic flux is constant. If you have a changing magnetic flux, you must use Faraday's Law (i.e. KVL is a special case of Faraday's Law for constant magnetic flux).

Ampere/Maxwell Law

Ampere's Law as written allows us to calculate the magnetic field due to an electric current, but we also know that a changing electric field produces a magnetic field. We can combine these two effects to obtain a more complete version of Ampere's Law. The contribution due to the penetrating current is known as the conduction current, and the contribution due to the changing electric field is known as the displacement current.

$$\oint_{\substack{\text{closed}\\ \text{loop}}} \vec{B} \bullet d\vec{l} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} = \mu_0 \varepsilon_0 \frac{d}{dt} \int_{\substack{\text{open}\\ \text{surface}}} \vec{E} \bullet d\vec{A}$$

Maxwell's Equations (complete version):

Gauss's Law $$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Gauss's Law for Magnetism $$\oint \vec{B} \bullet d\vec{A} = 0$$

Faraday's Law $$\oint_{\substack{\text{closed}\\ \text{loop}}} \vec{E} \bullet d\vec{l} = -\frac{d}{dt} \int_{\substack{\text{open}\\ \text{surface}}} \vec{B} \bullet d\vec{A}$$

Ampere/Maxwell Law $$\oint_{\substack{\text{closed}\\ \text{loop}}} \vec{B} \bullet d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$