AP-C Objectives (from College Board Learning Objectives for AP Physics

- 1. Newton's Law of Universal Gravitation
  - a. Determine the force that one spherically symmetrical mass exerts on another.
  - b. Determine the strength of the gravitational field at a specified point outside a spherically symmetrical mass.
  - c. Describe the gravitational force inside and outside a uniform sphere, and calculate how the field at the surface depends on the radius and density of the sphere.
- 2. Orbits of Planets and Satellites
  - a. Circular Orbits
    - i. Recognize that motion is independent of an object's mass.
    - ii. Describe qualitatively how velocity, period, and centripetal acceleration depend upon radius.
    - iii. Derive expressions for the velocity and period of revolution.
    - iv. Derive Kepler's Third Law for the case of circular orbits.
    - v. Derive and apply the relations among kinetic energy, potential energy, and total energy.
  - b. Elliptical Orbits
    - i. State Kepler's Laws of Planetary Motion and use them to describe the motion of an object in orbit.
    - ii. Apply conservation of angular momentum to determine the velocity and radial distance at any point in orbit.
    - iii. Apply energy conservation in analyzing the motion of an object that is projected straight up from a planet's surface or projected toward the planet from far above the surface.

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Newton's Law of Universal Gravitation

Any two masses exert an attractive force on each other.

$$\vec{F}_g = -\frac{Gm_1m_2}{r^2}\hat{r} \qquad G = 6.67\times10^{-11}\ {}^{N\bullet m^2}\!/_{kg^2}$$

Universal Gravitational Constant

Inverse Square Law

Because of the $1/r^2$ relationship, this is called an inverse square law. If the separation between the masses is doubled, for example, the force is quartered.

Example: Gravitation Force Between Earth and Sun

Find the magnitude of the gravitational force exerted on the Earth by the sun given the mass of the Earth ($m_e=6\times10^{24}$ kg), the mass of the sun ($m_s=2\times10^{30}$ kg), and the distance between them ($r=1.5\times10^{11}$ m).

$$\vec{F}_g = -\frac{Gm_1m_2}{r^2}\hat{r} \rightarrow \left|\vec{F}_g\right| \approx \frac{\left(6.67\times10^{-11}\ {}^{N\bullet m^2}\!/_{kg^2}\right)(6\times10^{24}\,kg)(2\times10^{30}\,kg)}{(1.5\times10^{11}\,m)^2} = 3.5\times10^{22}\,N$$

Gravitational Field Strength

Gravity is a non-contact force, known as a **field force.** Its effects are observed without the two objects coming into contact with each other. A gravitational field describes the gravitational force a mass would feel when placed at a particular point in space. The strength of the gravitational force on a test object in space can be represented by a vector at the position of the object. The denser the vectors, the stronger the force, the stronger the gravitational field.

$$\left|\vec{F}_g\right| = mg = \frac{Gm_1m_2}{r^2} \rightarrow g = \frac{Gm}{r^2}$$

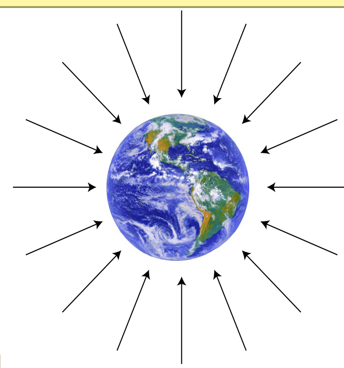
Therefore, the gravitational field strength, g, at any point in space is equal to the universal gravitational constant, G, times the mass of the object causing the field, divided by the square of the distance between the objects. The units of gravitational field strength, N/kg, are equivalent to the units of acceleration m/s^2. Try to calculate the gravitational field strength on the surface of Earth, where the distance from the center of the Earth to the Earth's surface is approximately 6378 km.

Example: Weight on Another Planet

If you weigh 600N on Earth, what will you weigh on a planet with twice the mass of Earth and half the radius of Earth?

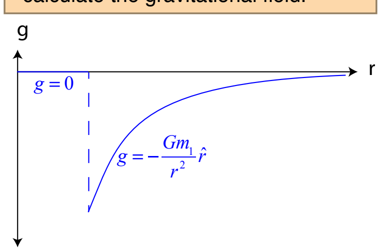
$$g = \frac{Gm}{r^2} \rightarrow g_p = \frac{Gm}{r^2} = \frac{G(2m_e)}{\left(\frac{r_e}{2}\right)^2} = 8\frac{Gm_e}{r_e^2} \rightarrow$$

$$g_p = 8g_e \rightarrow mg_p = 8mg_e = 8(600N) = 4800N$$



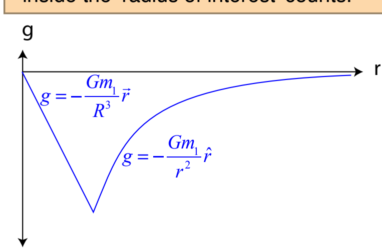
Gravitational Field of a Hollow Shell

Inside a hollow sphere, the gravitational field is 0. Outside a hollow sphere, you can treat the sphere as if it's entire mass was concentrated at the center, and then calculate the gravitational field.



Gravitational Field Inside a Solid Sphere

Outside a solid sphere, treat the sphere as if all the mass is at the center of the sphere. Inside the sphere, treat the sphere as if the mass inside the radius is all at the center. Only the mass inside the 'radius of interest' counts.



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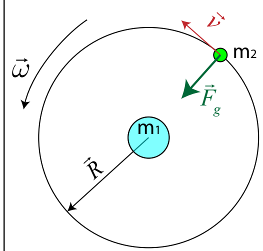
Analysis of Circular Orbits



▼ Velocity of a Satellite in Orbit

Gravity is the centripetal force for an object m_2 traveling in a circular orbit about object m_1 with velocity v. Utilize Newton's 2nd Law to solve for the velocity of m_2.

$$F_{net_c} = \vec{F}_g = m_2\vec{a}_c \xrightarrow[a_c=\frac{v^2}{R}]{F_g=\frac{Gm_1m_2}{R^2}} \frac{Gm_1m_2}{R^2} = m_2\frac{v^2}{R} \rightarrow$$

$$\frac{Gm_1}{R} = v^2 \rightarrow v = \sqrt{\frac{Gm_1}{R}}$$

Note: velocity of satellite is independent of satellite's mass!

▼ Period and Frequency

Period (T) is the time it takes for the satellite to make one complete revolution, or travel one circumference of its circular path. You can solve for the period using the velocity defined above. Frequency (f) is the number of revolutions per second (Hz), and is the inverse of period (f=1/T).

$$2\pi R = vT \rightarrow T = \frac{2\pi R}{v} \xrightarrow{v=\sqrt{\frac{Gm_1}{R}}}$$

$$T = \frac{2\pi R\sqrt{R}}{\sqrt{Gm_1}} \rightarrow T^2 = \frac{4\pi^2R^3}{Gm_1}$$

This is Kepler's 3rd Law of Planetary Motion for a Circular Orbit!

▼ Total Mechanical Energy for a Circular Orbit

Total mechanical energy is the sum of kinetic and potential energies.

$$E = K+U \xrightarrow[U=\frac{-Gm_1m_2}{R}]{K=\frac{1}{2}mv^2} E=\tfrac{1}{2}m_2v^2 - \frac{Gm_1m_2}{R} \xrightarrow[v^2=\frac{Gm_1}{R}]{v=\sqrt{\frac{Gm_1}{R}}}$$

$$E=\tfrac{1}{2}m_2\frac{Gm_1}{R} - \frac{Gm_1m_2}{R} \rightarrow E = \frac{-Gm_1m_2}{2R}$$

▼ Escape Velocity

Escape velocity is the velocity required to completely escape the influence of gravity (occurs at an infinite distance from a mass). You can find escape velocity by recognizing that the gravitational potential energy is zero when an object has complete escaped gravity's influence, and it has no leftover kinetic energy (v=0).

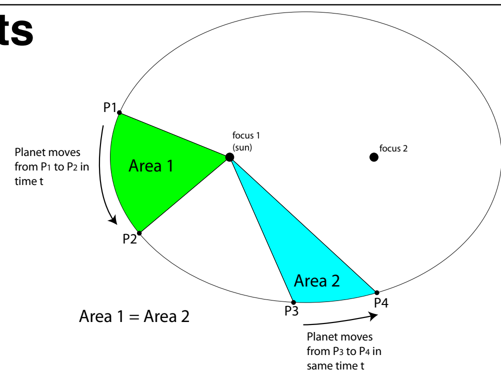
$$E=K+U=0 \rightarrow 0=\tfrac{1}{2}m_2v^2-\frac{Gm_1m_2}{R} \rightarrow \tfrac{1}{2}m_2v^2 = \frac{Gm_1m_2}{R}\rightarrow$$

$$v^2 = \frac{2Gm_1m_2}{m_2R} = \frac{2Gm_1}{R} \rightarrow v_{escape} = \sqrt{\frac{2Gm_1}{R}}$$

Analysis of Elliptical Orbits

▼ Kepler's Laws of Planetary Motion

1. The orbits of planetary bodies are ellipses with the sun at one of the two foci of the ellipse.
2. If you were to draw a line from the sun to the orbiting body, the body would sweep out equal areas along the ellipse in equal amounts of time.
3. The ratio of the squares of the periods of two planets is equal to the ratio of the cubes of their semi-major axes. The ratio of the squares of the periods to the cubes of their semi-major axes is referred to as Kepler's Constant.



▼ Total Mechanical Energy for an Elliptical Orbit

Total mechanical energy is the sum of kinetic and potential energies.

$$E=K+U=\tfrac{1}{2}mv^2 - \frac{Gm_1m_2}{r} = -\frac{Gm_1m_2}{2a}$$

▼ Velocity and Radius of an Elliptical Orbit

Analyze the system from reference point P.

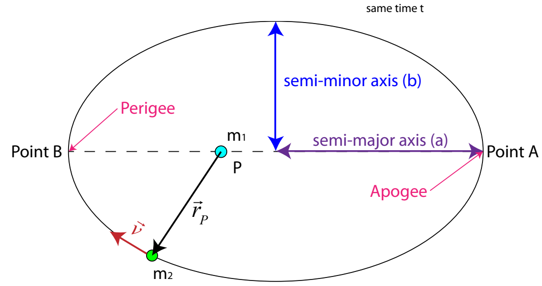
$$\frac{d\vec{L}_P}{dt} = \vec{\tau}_P \xrightarrow[\text{only force is gravity through point P}]{\tau_P=0} \frac{d\vec{L}_P}{dt}=0\rightarrow$$

Angular momentum must be conserved!

$$|L_P| = m_2v_Pr_P\sin\theta_P = m_2v_Ar_A\sin\theta_A \xrightarrow[\sin 90^\circ=1]{\theta_P=\theta_A=90^\circ}$$

$$v_Pr_P = v_Ar_A$$

You have a relationship for velocity and radius for any point in orbit!



▼ Sample Problem: Rocket Launched Vertically

A rocket is launched vertically from the surface of the Earth with an initial velocity of 10 km/s. What maximum height does it reach, neglecting air resistance? Note that the mass of the Earth (m_1) is 6×10^{24} kg and the radius of the Earth (R_E) is 6.37×10^6 m. You may not assume that the acceleration due to gravity is a constant.

$$E=K_i+U_i=K_f+U_f \rightarrow \tfrac{1}{2}m_2v^2 + \frac{-Gm_1m_2}{R_E} = 0 + \frac{-Gm_1m_2}{R}\rightarrow$$

$$v^2 - \frac{2Gm_1}{R_E} = \frac{-2Gm_1}{R} \rightarrow \frac{1}{R} = \frac{-v^2}{2Gm_1}+\frac{2Gm_1}{2Gm_1R_E}\rightarrow \frac{1}{R}=\frac{-v^2}{2Gm_1}+\frac{1}{R_E}\rightarrow$$

$$\frac{1}{R} = \frac{-(10{,}000\,\tfrac{m}{s})^2}{2(6.67\times10^{-11}\,\tfrac{N\cdot m^2}{kg^2})(6\times10^{24}\,kg)} + \frac{1}{(6.37\times10^6\,m)} \rightarrow$$

$$R = 3.12\times10^7\,m$$

height above surface of the Earth is the distance of the rocket from the center of the Earth to the radius of the Earth.

$$h = R - R_E = 3.12\times10^7\,m - 6.37\times10^6\,m = 2.48\times10^7\,m$$