AP-C Objectives (from College Board Learning Objectives for AP Physics)

▼ 1. Forces on moving charges in magnetic fields
    a. Calculate the magnitude and direction of the force in terms of q, v, and B, and explain why the magnetic force can perform no work.
    b. Deduce the direction of a magnetic field from information about the forces experienced by charged particles moving through that field.
    c. Describe the paths of charged particles moving in uniform magnetic fields.
    d. Derive and apply the formula for the radius of the circular path of a charge that moves perpendicular to a uniform magnetic field.
    e. Describe under what conditions particles will move with constant velocity through crossed electric and magnetic fields.

▼ 2. Forces on current-carrying wires in magnetic fields
    a. Calculate the magnitude and direction of the force on a straight segment of current-carrying wire in a uniform magnetic field.
    b. Indicate the direction of magnetic forces on a current-carrying loop of wire in a magnetic field, and determine how the loop will tend to rotate as a consequence of these forces.
    c. Calculate the magnitude and direction of the torque experienced by a rectangular loop of wire carrying a current in a magnetic field.

▼ 3. Fields of long current-carrying wires
    a. Calculate the magnitude and direction of the field at a point in the vicinity of such a wire.
    b. Use superposition to determine the magnetic field produced by two long wires.
    c. Calculate the force of attraction or repulsion between two long current-carrying wires.

▼ 4. Biot-Savart law and Ampere's law
    ▼ a. Students should understand the Biot-Savart Law, so they can:
        i. Deduce the magnitude and direction of the contribution to the magnetic field made by a short straight segment of current-carrying wire.
        ii. Derive and apply the expression for the magnitude of B on the axis of a circular loop of current.
    ▼ b. Students should understand the statement and application of Ampere's Law in integral form, so they can:
        i. State the law precisely.
        ii. Use Ampere's law, plus symmetry arguments and the right-hand rule, to relate magnetic field strength to current for planar or cylindrical symmetries.
    c. Students should be able to apply the superposition principle so they can determine the magnetic field produced by combinations of the configurations listed above.

Forces on Moving Charges in Magnetic Fields

Objectives

1. Calculate the magnitude and direction of the force in terms of **q**, v, and B, and explain why the magnetic force can perform no work.
2. Deduce the direction of a magnetic field from information about the forces experienced by charged particles moving through that field.
3. Describe the paths of charged particles moving in uniform magnetic fields.
4. Derive and apply the formula for the radius of the circular path of a charge that moves perpendicular to a uniform magnetic field.
5. Describe under what conditions particles will move with constant velocity through crossed electric and magnetic fields.

▼ Units of Magnetic Field

Standard (SI) units of magnetic field are Tesla

1 Tesla (1 T) = 1 N•s/(C•m)

1 Tesla is a very strong magnetic field

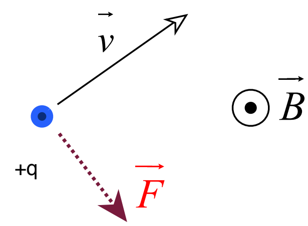
More common non-SI unit is the Gauss

1 Gauss = 10^{-4} Tesla

Earth's magnetic field strength ≈ 0.5 Gauss

▼ Magnetic Forces Cannot Perform Work on Moving Charges

Force is always perpendicular to velocity, therefore the magnetic force on a moving charge is never applied in the direction of the displacement, therefore a magnetic force can do no work on a moving charge (but it can change its direction).



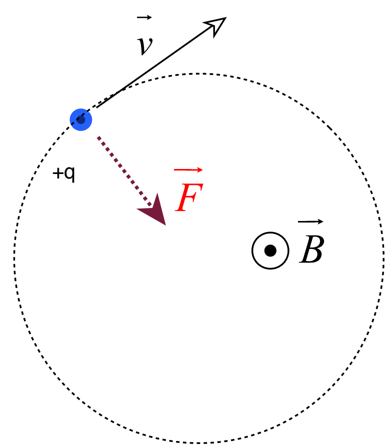
$$\boxed{F_M = q\vec{v}\times\vec{B}}$$

$$\boxed{\left|\vec{F}_M\right| = qvB\sin\theta}$$

▼ Forces on Moving Charges in Magnetic Fields

Magnetic force cannot perform work on a moving charge

Magnetic force can change its direction (moving it in a circle if FM is constant.)



$$\left|\vec{F}_M\right| = qvB\sin\theta \xrightarrow[\sin 90^\circ=1]{\theta=90^\circ}$$

$$\left|\vec{F}_M\right| = qvB$$

$$F_M = F_C \rightarrow qvB = \frac{mv^2}{r} \rightarrow$$

$$r = \frac{mv}{qB} \leftarrow \text{momentum}$$

▼ Total Force on a Moving Charged Particle (E field and B Field)

E field can do work on a moving charge

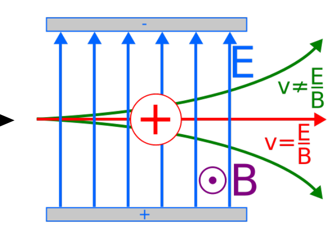
B field can never do work on a moving charge

Lorentz Force

$$\vec{F}_{TOT} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

▼ The Velocity Selector

A charged particle in crossed E and B fields can undergo constant velocity motion if v, B, and E are all selected perpendicular to each other. Then, if v=E/B, the particle can travel through the selector without any deflection, while particles with any other velocity are diverted.

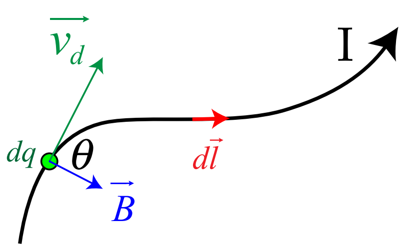


Objectives

1. Calculate the magnitude and direction of the force on a straight segment of current-carrying wire in a uniform magnetic field.
2. Indicate the direction of magnetic forces on a current-carrying loop of wire in a magnetic field, and determine how the loop will tend to rotate as a consequence of these forces.
3. Calculate the magnitude and direction of the torque experienced by a rectangular loop of wire carrying a current in a magnetic field.

▼ Forces on Current-Carrying Wires

Moving charges in magnetic fields experience forces.
Current in a wire is just the flow of positive charges. If charges are moving perpendicular to magnetic fields, they experience a force which is applied to the wire.



$$\vec{F}_B = q(\vec{v}\times\vec{B}) \rightarrow d\vec{F}_B = dq(\vec{v}_d \times \vec{B}) \xrightarrow[dq=Idt]{I=\frac{dq}{dt}}$$

$$d\vec{F}_B = Idt(\vec{v}_d \times \vec{B}) \xrightarrow[\vec{v}_d dt = d\vec{l}]{\vec{v}_d=\frac{d\vec{l}}{dt}} d\vec{F}_B = I(d\vec{l}\times\vec{B}) \rightarrow$$

$$\int d\vec{F}_B = \int I(d\vec{l}\times\vec{B}) \rightarrow$$

$$\vec{F}_B = \int I(d\vec{l}\times\vec{B})$$

Example: A straight wire of length 1 m carries a current of 100A through a magnetic field of 1 Tesla. Find the force on the wire.

⟶ I=100A

ⓧ B=1 Tesla

$$\int d\vec{F}_B = \int I(d\vec{l}\times\vec{B}) \rightarrow F_B = IlB\sin\theta \rightarrow$$

$$F_B = IlB \xrightarrow[l=1m \;\; B=1T]{I=100A} F_B = (100A)(1m)(1T) \rightarrow$$

$$F_B = 100N$$

▼ Direction of Force

Determine the direction of the force on the wire using the 3rd right hand rule.

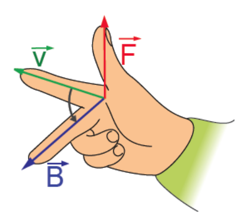
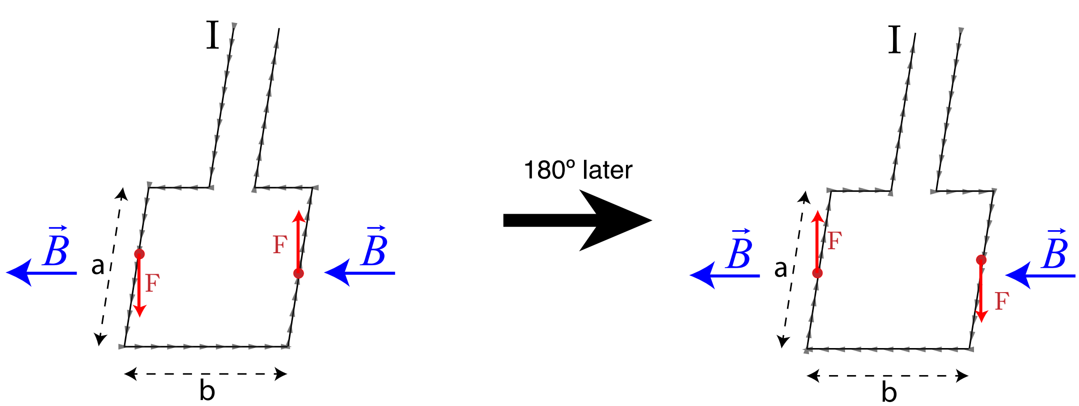


Illustration Courtesy of David Crochet

Electric Motors



$F = IaB \qquad F_{net} = 0$

$\tau_{net} = r\times F = \frac{b}{2}IaB + \frac{b}{2}IaB = IabB$

Creates counter-clockwise rotation

Net torque is reversed
To keep motor moving, you need to either reverse the current direction with a commutator or turn off the current in this stage.

Fields of long current-carrying wires

Objectives

1. Calculate the magnitude and direction of the field at a point in the vicinity of a long current-carrying wire.
2. Use superposition to determine the magnetic field produced by two long wires.
3. Calculate the force of attraction or repulsion between two long current-carrying wires.

▼ Magnetic Fields produced by long straight current-carrying wires

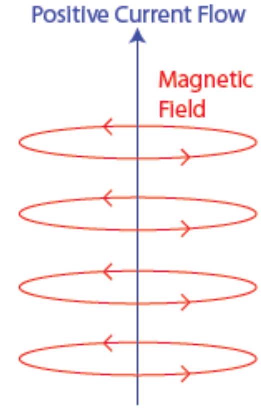
Moving charges create magnetic fields.
Current is moving positive charges, therefore current-carrying wires create magnetic fields.
Use 1st Right Hand Rule to find direction of magnetic field.
If you have two wires, determine magnetic field from each and add them up using superposition principle.
These fields may interact with other moving charges, so current-carrying wires can create forces of attraction or repulsion between themselves.

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\mu_0 = 4\pi \times 10^{-7}\ {}^{T\bullet m}/_{A}$$

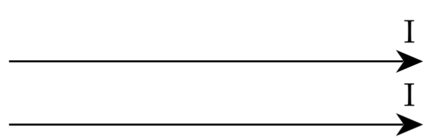
▼ 1st Right Hand Rule

"Hold" wire with your right hand, with thumb in the direction of positive current flow. Your fingers "wrap around" the wire in the direction of the magnetic field.

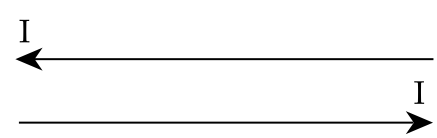


▼ Force Between Parallel Current-Carrying Wires

Use right hand rules to determine force between parallel current-carrying wires.
Find magnetic field due to first wire. Draw it.
Find direction of force on 2nd wire due to current in second wire. Force on 1st wire will be equal and opposite (Newton's 3rd)



Wires Attract Each Other



Wires Repel Each Other

▼ Gauss's Law for Magnetism

You can never draw a closed surface with any net magnetic flux because there are no magnetic monopoles.
This is the basis of Gauss's Law for Magnetism (Maxwell's 2nd Equation)

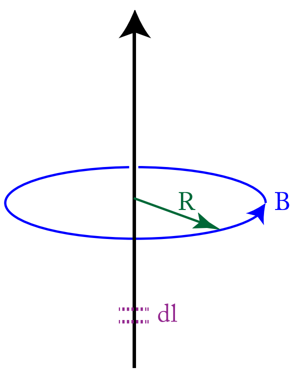
$$\Phi_{B_{net}} = \oint \vec{B} \bullet d\vec{A} = 0$$

Objectives

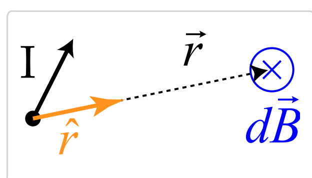
1. Understand Biot-Savart Law
   a. Deduce the magnitude and direction of the contribution to the magnetic field made by a short straight segment of current-carrying wire.
   b. Derive and apply the expression for the magnitude of B on the axis of a circular loop of current.

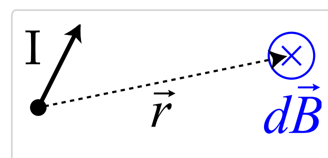
Biot-Savart Law

"Brute Force" method of finding the magnetic field due to a length of current-carrying wire

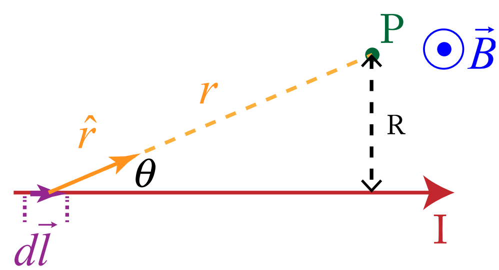


$$d\vec{B} = \frac{\mu_0 I (d\vec{l} \times \hat{r})}{4\pi r^2} \xrightarrow{\hat{r} = \frac{\vec{r}}{r}} d\vec{B} = \frac{\mu_0 I (d\vec{l} \times \vec{r})}{4\pi r^3}$$





Example: Derive the B field due to a long straight current-carrying wire



$$d\vec{B} = \frac{\mu_0 I (d\vec{l} \times \hat{r})}{4\pi r^2} \xrightarrow{|d\vec{l} \times \hat{r}| = dx \sin\theta}$$

$$dB = \frac{\mu_0 I dx \sin\theta}{4\pi r^2} \xrightarrow[\sin\theta = \frac{R}{\sqrt{x^2+R^2}}]{r^2 = x^2 + R^2}$$

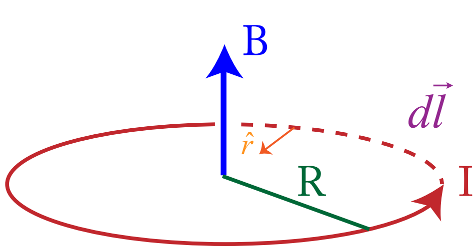
$$dB = \frac{\mu_0 I dx R}{4\pi (x^2+R^2)^{\frac{3}{2}}} \rightarrow \int dB = B = \int_{x=-\infty}^{x=\infty} \frac{\mu_0 I dx R}{4\pi (x^2+R^2)^{\frac{3}{2}}} \rightarrow$$

$$B = \frac{\mu_0 I R}{4\pi} \int_{-\infty}^{\infty} \frac{dx}{(x^2+R^2)^{\frac{3}{2}}} \xrightarrow{\text{Table of Integrals}}$$

$$B = \frac{\mu_0 I R}{4\pi} \left(\frac{x}{R^2 (R^2+x^2)^{\frac{1}{2}}} \right) \Bigg|_{-\infty}^{\infty} = \frac{\mu_0 I R}{4\pi} \left(\frac{\infty}{R^2 (R^2+\infty^2)^{\frac{1}{2}}} - \frac{-\infty}{R^2 (R^2+(-\infty)^2)^{\frac{1}{2}}} \right) \rightarrow$$

$$B = \frac{\mu_0 I R}{4\pi} \left(\frac{1}{R^2} + \frac{1}{R^2} \right) = \frac{\mu_0 I R}{4\pi} \left(\frac{2}{R^2} \right) \rightarrow B = \frac{\mu_0 I}{2\pi R}$$

Example: Derive the magnetic field due to a current loop



$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi r^2} \int_{wire} d\vec{l} \times \hat{r} \xrightarrow[\sin\theta = \sin 90° = 1]{d\vec{l} \times \hat{r} = dl \sin\theta}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi r^2} \int_{wire} d\vec{l} = \frac{\mu_0 I}{4\pi r^2} (2\pi r) \rightarrow \vec{B} = \frac{\mu_0 I}{2r}$$

Biot-Savart Law (Alternate)

Objectives

▼ 1. Understand Biot-Savart Law

a. Deduce the magnitude and direction of the contribution to the magnetic field made by a short straight segment of current-carrying wire.

b. Derive and apply the expression for the magnitude of B on the axis of a circular loop of current.

▼ Problem Solving Steps

1. Look for symmetries and simplifications.
2. Define your five quantities (note that prime mark indicates source of magnetic field)

$\vec{r}, \vec{r}\,', d\vec{I}, \vec{\mathbb{R}}, \mathbb{R}$

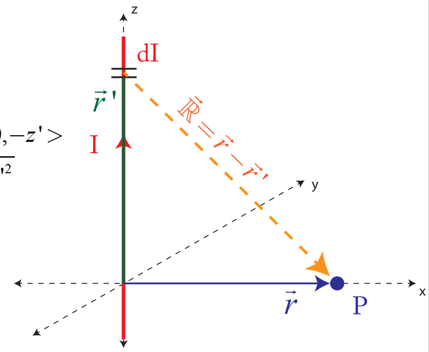
3. Set up your integral.
4. Integrate (may require computer assistance for all but the simplest cases)

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{d\vec{I}\times\hat{\mathbb{R}}}{\mathbb{R}^2} = \frac{\mu_0}{4\pi}\frac{d\vec{I}\times\vec{\mathbb{R}}}{\mathbb{R}^3}$$

$$\vec{\mathbb{R}} = \vec{r} - \vec{r}\,'$$

Example: Derive the B field due to a long straight current-carrying wire. Note cylindrical symmetry, common sense tells us only positional dependence will be radial.

$$\vec{r} = <x,0,0>$$
$$\vec{r}\,' = <0,0,z'>$$
$$d\vec{I} = <0,0,Idz'>$$
$$\vec{\mathbb{R}} = \vec{r} - \vec{r}\,' = <x,0,-z'>$$
$$\mathbb{R} = \left|\vec{\mathbb{R}}\right| = \sqrt{x^2 + z'^2}$$



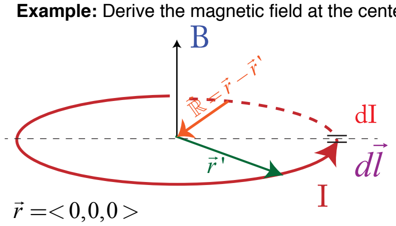
Once five items are defined, you can integrate:

$$\vec{B} = \int \frac{\mu_0}{4\pi}\frac{d\vec{I}\times\vec{\mathbb{R}}}{\mathbb{R}^3} \xrightarrow{d\vec{I}\times\vec{\mathbb{R}} = <0,Ixdz',0>}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{Ixdz'\,\hat{j}}{(x^2+z'^2)^{\frac{3}{2}}} = \frac{\mu_0 Ix\hat{j}}{4\pi}\int_{-\infty}^{\infty}\frac{dz'}{(x^2+z'^2)^{\frac{3}{2}}} \xrightarrow{\text{Table of Integrals}}$$

$$\vec{B} = \frac{\mu_0 Ix\hat{j}}{4\pi}\frac{2}{x^2} = \frac{\mu_0 I}{2\pi x}\hat{j} \xrightarrow{\text{Symmetry}} \left|\vec{B}\right| = \frac{\mu_0 I}{2\pi r}$$

Example: Derive the magnetic field at the center of a current loop of radius R.



Note: Symmetries indicate all the magnetic field will be in the positive z direction (toward the top of the page)

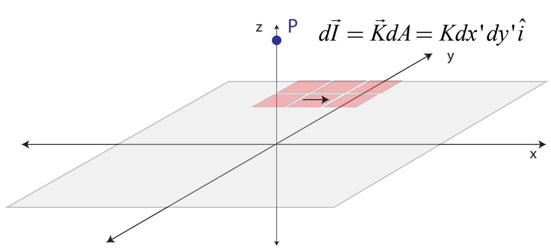
$$\vec{r} = <0,0,0>$$
$$\vec{r}\,' = <R',0,0>$$
$$d\vec{I} = <0,Idl',0>$$
$$\vec{\mathbb{R}} = \vec{r} - \vec{r}\,' = <-R',0,0>$$
$$\mathbb{R} = \left|\vec{\mathbb{R}}\right| = R$$

$$\vec{B} = \int \frac{\mu_0}{4\pi}\frac{d\vec{I}\times\vec{\mathbb{R}}}{\mathbb{R}^3} \xrightarrow[d\vec{I}\times\vec{\mathbb{R}} = <0,0,IR'dl'> = IR'dl'\hat{k}]{d\vec{I}\times\vec{\mathbb{R}} = <0,Idl',0>\times<-R',0,0>}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{IR'}{R^3}\hat{k}\int_{loop} dl' = \frac{\mu_0}{4\pi}\frac{I}{R^2}\hat{k}(2\pi R) = \frac{\mu_0 I}{2R}\hat{k}$$

Example: Derive the magnetic field at some distance from an infinite sheet of charge.

Strategy: Break up sheet into infinitessimally small squares of area dA, each with a current per perpendicular length of σv, which we can define as the aerial current density K.



$$d\vec{I} = \vec{K}dA = Kdx'dy'\hat{i}$$

$$d\vec{I}\times\vec{\mathbb{R}} = (Kdx'dy'\hat{i})\times(-x'\hat{i} - y'\hat{j} + z\hat{k}) \rightarrow$$
$$d\vec{I}\times\vec{\mathbb{R}} = (Kdx'dy'\hat{i})\times(-y'\hat{j}) + (Kdx'dy'\hat{i})\times(z\hat{k}) \rightarrow$$
$$d\vec{I}\times\vec{\mathbb{R}} = -Kzdx'dy'\hat{j} - Ky'dx'dy'\hat{k} \rightarrow$$
$$d\vec{I}\times\vec{\mathbb{R}} = <0,-Kzdx'dy',-Ky'dx'dy'>$$

$$\vec{r} = <0,0,z> = z\hat{k}$$
$$\vec{r}\,' = <x',y',0> = x'\hat{i} + y'\hat{j}$$
$$d\vec{I} = <Kdx'dy',0,0> = Kdx'dy'\hat{i}$$
$$\vec{\mathbb{R}} = \vec{r} - \vec{r}\,' = <-x',-y',z> = -x'\hat{i} - y'\hat{j} + z\hat{k}$$
$$\mathbb{R} = \left|\vec{\mathbb{R}}\right| = \sqrt{x'^2 + y'^2 + z^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{d\vec{I}\times\hat{\mathbb{R}}}{\mathbb{R}^2} = \frac{\mu_0}{4\pi}\frac{d\vec{I}\times\vec{\mathbb{R}}}{\mathbb{R}^3} \rightarrow \vec{B} = \iint_{\infty}\frac{\mu_0}{4\pi}\frac{-Kzdx'dy'\hat{j} - Ky'dx'dy'\hat{k}}{\sqrt{x'^2+y'^2+z^2}} \rightarrow$$

$$\vec{B} = \iint_{\infty}\frac{\mu_0}{4\pi}\frac{-Kzdx'dy'\hat{j}}{\sqrt{x'^2+y'^2+z^2}} \xrightarrow[\text{or Computer}]{\text{Table of Integrals}} \vec{B} = \frac{-\mu_0 K}{2}\frac{z}{|z|}\hat{j}$$

Note: z-component (in **k** direction) must be 0 since magnetic fields must make complete loops (divergence of magnetic field is zero)

Objectives

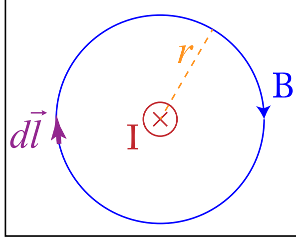
1. Understand Ampere's Law
   a. State the law precisely.
   b. Use Ampere's law, plus symmetry arguments and the right-hand rule, to relate magnetic field strength to current for planar or cylindrical symmetries.
2. Apply the superposition principle to determine the magnetic field produced by combinations of Biot-Savart and Ampere's laws configurations listed above.

Ampere's Law

Elegant method of finding magnetic field in situations of symmetry.

$$\oint_{\substack{closed\\loop}} \vec{B}\bullet d\vec{l} = \mu_0 I_{penetrating}$$

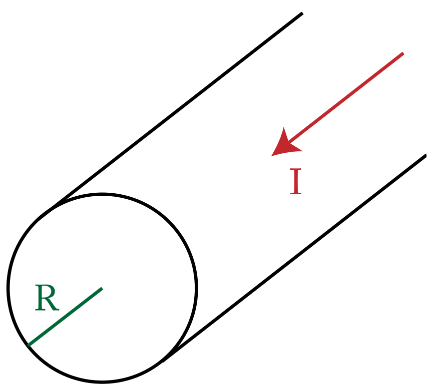
Example: Find magnetic field strength due to a current-carrying wire



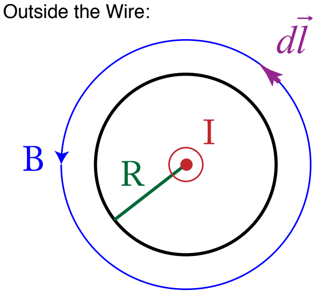
$$\oint_{\substack{closed\\loop}} \vec{B}\bullet d\vec{l} = \mu_0 I_{penetrating} \rightarrow$$

$$B(2\pi r) = \mu_0 I \rightarrow B = \frac{\mu_0 I}{2\pi r}$$

Example: Find the magnetic field everywhere for a current-carrying wire (inside and outside the wire)



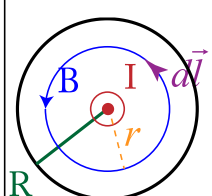
Outside the Wire:



$$\oint_{\substack{closed\\loop}} \vec{B}\bullet d\vec{l} = \mu_0 I_{penetrating} \rightarrow$$

$$B(2\pi r) = \mu_0 I \rightarrow B = \frac{\mu_0 I}{2\pi r}$$

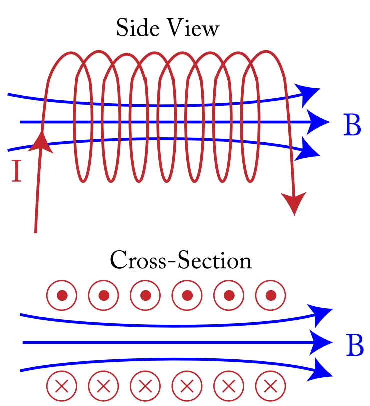
Inside the Wire:



$$\oint_{\substack{closed\\loop}} \vec{B}\bullet d\vec{l} = \mu_0 I_{penetrating} \rightarrow$$

$$B(2\pi r) = \mu_0 \left(\frac{\pi r^2}{\pi R^2}\right) I \rightarrow B = \frac{\mu_0 I r}{2\pi R^2}$$

Example: Calculate the magnetic field in the middle of a solenoid (i.e. Slinky) using Ampere's Law.



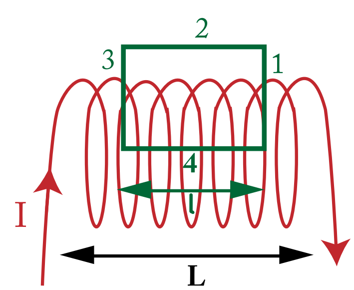
Assume B outside solenoid is 0.
N loops of wire.

First choose a rectangular closed loop for application of Ampere's Law.

Path 1 and 3: integral of B•dl is 0 since the angle between B and dl is 90°.

Path 2 integral is 0 with assumption B is close to 0 outside solenoid

Integrate through Path 4



$$\oint_{\substack{closed\\loop}} \vec{B}\bullet d\vec{l} = \mu_0 I_{penetrating} \xrightarrow{I_{pen}=\frac{l}{L}NI}$$

$$Bl = \tfrac{l}{L} N \mu_0 I \rightarrow B = \tfrac{N}{L}\mu_0 I$$