AP-C Objectives (from College Board Learning Objectives for AP Physics)

▼ 1. Impulse and Momentum

- a. Relate mass, velocity, and linear momentum for a moving object, and calculate the total linear momentum of a system of objects.
- b. Relate impulse to the change in linear momentum and the average force acting on an object.
- c. State and apply the relations between linear momentum and center-of-mass motion for a system of particles.
- d. Calculate the area under a force versus time graph and relate it to the change in momentum of an object.
- e. Calculate the change in momentum of an object given a function $F(t)$ for the net force acting on the object.

▼ 2. Conservation of Linear Momentum, Collisions

- a. Explain how linear momentum conservation follows as a consequence of Newton's Third Law for an isolated system.
- b. Identify situations in which linear momentum, or a component of the linear momentum vector, is conserved.
- c. Apply linear momentum conservation to two-dimensional collisions.
- d. Analyze situations in which two or more objects are pushed apart by a spring or other agency, and calculate how much energy is released in such a process.

▼ 3. Center of Mass

- a. Identify by inspection the center of mass of a symmetrical object.
- b. Locate the center of mass of a system consisting of two such objects.
- c. Use integration to find the center of mass of a thin rod of non-uniform density.
- d. Apply the relation between center-of-mass velocity and linear momentum, and between center-of-mass acceleration and net external force for a system of particles.
- e. Define center of gravity and use this concept to express the gravitational potential energy of a rigid object in terms of the position of its center of mass.

Impulse and Momentum

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Momentum

Momentum is a vector describing how difficult it is to stop a moving object.
Total momentum is the sum of individual momenta.

$$\vec{p} = m\vec{v}$$

Relationship between Force and Change in Momentum

$$\vec{F} = m\vec{a} \xrightarrow{a=\frac{d\vec{v}}{dt}} \vec{F} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) \xrightarrow{\vec{p}=m\vec{v}} \vec{F} = \frac{d\vec{p}}{dt}$$

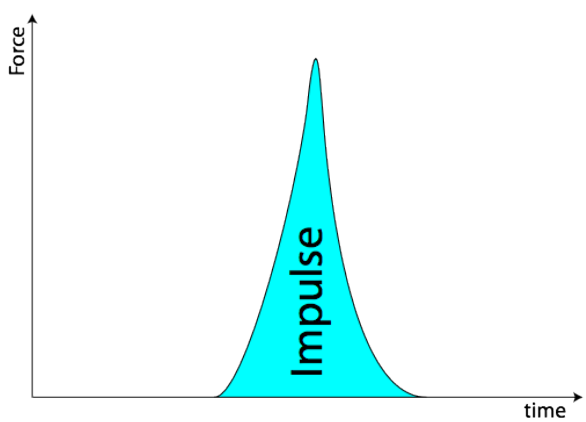
Force and Momentum

If a force acts on a particle, it changes the particle's momentum.
If a particle's momentum changes, a force must have acted on it.

Impulse-Momentum Theorem

Impulse (J) changes momentum.

$$\vec{F} = \frac{d\vec{p}}{dt} \rightarrow \vec{F}dt = d\vec{p} \rightarrow \int_0^t \vec{F}\,dt = \int_0^t dp \rightarrow \int_0^t \vec{F}\,dt = \vec{J} = \vec{F}\Delta t = \Delta\vec{p}$$



Sample Problem

The momentum of an object as a function of time is given by p=kt², where k is a constant. What is the equation for the force causing this motion?

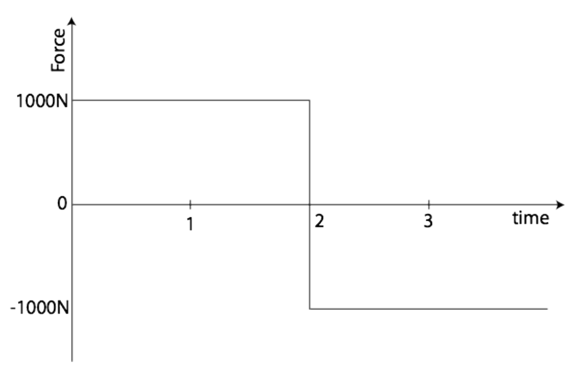
$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(kt^2) = 2kt$$

Sample Problem

The graph indicates the force on a truck of mass 2000kg as a function of time. In the interval of 0 to 3 seconds, determine the change in the truck's velocity.

$$\vec{J} = \Delta p = F\Delta t \rightarrow (2000N \bullet s) - (1000N \bullet s) = m\Delta v \rightarrow$$

$$\Delta v = \frac{1000N \bullet s}{2000kg} = 0.5 \,{}^{m}\!/_{s}$$



Conservation of Linear Momentum

AP-C Objectives (from College Board Learning Objectives for AP Physics)

▼ 1. Conservation of Linear Momentum, Collisions

- a. Explain how linear momentum conservation follows as a consequence of Newton's Third Law for an isolated system.
- b. Identify situations in which linear momentum, or a component of the linear momentum vector, is conserved.
- c. Apply linear momentum conservation to two-dimensional collisions.
- d. Analyze situations in which two or more objects are pushed apart by a spring or other agency, and calculate how much energy is released in such a process.

▼ Conservation of Linear Momentum

In an isolated system, where no external forces act, momentum is always conserved. Put more simply, in any closed system, the total momentum of the system remains constant. This is a direct outcome of Newton's 3rd Law of Motion.

▼ Collisions and Explosions

In the case of a collision or explosion (an event), if you add up the individual momentum vectors of all the objects before the event, you'll find that they are equal to the sum of the momentum vectors of the objects after the event. Written mathematically, the law of conservation of momentum states that $p_{initial} = p_{final}$.

Creating Momentum Tables

1. Identify all objects in the system. List them vertically down the left-hand column.
2. Determine the momenta of the objects before the event. Use variables for any unknowns.
3. Determine the momenta of the objects after the event. Use variables for any unknowns.
4. Add up all the momenta from before the event and set them equal to the momenta after the event.
5. Solve your resulting equation for any unknowns.

▼ Sample Problem

A 2000-kg car traveling at 20 m/s collides with a 1000-kg car at rest at a stop sign. If the 2000-kg car has a velocity of 6.67 m/s after the collision, find the velocity of the 1000-kg car after the collision.

Call the 2000-kg car Car A, and the 1000-kg car Car B. You can then create a momentum table as shown below:

Objects	Momentum Before (kg·m/s)	Momentum After (kg·m/s)
Car A	2000×20=40,000	2000×6.67=13,340
Car B	1000×0=0	$1000\times v_B = 1000v_B$
Total	40,000	$13{,}340 + 1000v_B$

Because momentum is conserved in any closed system, the total momentum before the event must be equal to the total momentum after the event.

$$40{,}000 = 13{,}340 + 1000v_B$$

$$v_B = \frac{40{,}000 - 13{,}340}{1000} = 26.7 \, \text{m/s}$$

Collisions in 2 Dimensions

AP-C Objectives (from College Board Learning Objectives for AP Physics)

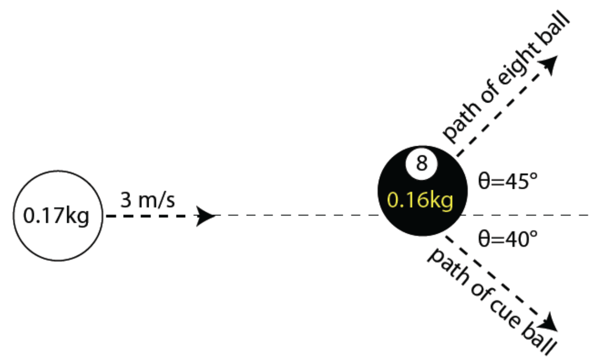
- 1. Conservation of Linear Momentum, Collisions
  - a. Apply linear momentum conservation to two-dimensional collisions.

Collisions in Multiple Dimensions

Momentum is independently conserved in X, Y, and Z dimensions.

Sample Problem — 2-D Collision

Bert strikes a cue ball of mass 0.17 kg, giving it a velocity of 3 m/s in the x-direction. When the cue ball strikes the eight ball (mass=0.16 kg), previously at rest, the eight ball is deflected 45 degrees from the cue ball's previous path, and the cue ball is deflected 40 degrees in the opposite direction. Find the velocity of the cue ball and the eight ball after the collision.



Start by making momentum tables for the collision, beginning with the x-direction. Since you don't know the velocity of the balls after the collision, call the velocity of the cue ball after the collision v_c, and the velocity of the eight ball after the collision v_8. Note that you must use trigonometry to determine the x-component of the momentum of each ball after the collision.

Objects	X-Momentum Before (kg·m/s)	X-Momentum After (kg·m/s)
Cue Ball	0.17×3=0.51	$0.17 \times v_c \times \cos(40°)$
Eight Ball	0	$0.16 \times v_8 \times \cos(45°)$
Total	0.51	$0.17 \times v_c \times \cos(40°) + 0.16 \times v_8 \times \cos(45°)$

Since the total momentum in the x-direction before the collision must equal the total momentum in the x-direction after, the collision, you can set the total before and total after columns equal:

$$0.51 \tfrac{kg\cdot m}{s} = (0.17kg)(\cos 40°)v_c + (0.16kg)(\cos 45°)v_8$$
$$0.51 \tfrac{kg\cdot m}{s} = (0.130kg)v_c + (0.113kg)v_8$$

Next, create a momentum table and algebraic equation for the conservation of momentum in the y-direction.

Objects	Y-Momentum Before (kg·m/s)	Y-Momentum After (kg·m/s)
Cue Ball	0	$0.17 \times v_c \times \sin(-40°)$
Eight Ball	0	$0.16 \times v_8 \times \sin(45°)$
Total	0	$0.17 \times v_c \times \sin(-40°) + 0.16 \times v_8 \times \sin(45°)$

$$0 = (0.17kg)(\sin -40°)v_c + (0.16kg)(\sin 45°)v_8$$
$$0 = (-0.109kg)v_c + (0.113kg)v_8$$

You now have two equations with two unknowns. To solve this system of equations, start by solving the y-momentum equation for v_c.

$$0 = (-0.109kg)v_c + (0.113kg)v_8$$
$$(0.109kg)v_c = (0.113kg)v_8$$
$$v_c = 1.04v_8$$

You can now take this equation for v_c and substitute it into the equation for conservation of momentum in the x-direction, effectively eliminating one of the unknowns, and giving a single equation with a single unknown.

$$0.51 \tfrac{kg\cdot m}{s} = (0.130kg)v_c + (0.113kg)v_8 \xrightarrow{v_c=1.04v_8}$$
$$0.51 \tfrac{kg\cdot m}{s} = (0.130kg)(1.04v_8) + (0.113kg)v_8$$
$$0.51 \tfrac{kg\cdot m}{s} = (0.248kg)v_8$$
$$v_8 = 2.06 \tfrac{m}{s}$$

Finally, solve for the velocity of the cue ball after the collision by substituting the known value for v_8 into the result of the y-momentum equation.

$$v_c = 1.04v_8 \xrightarrow{v_8=2.06\frac{m}{s}}$$
$$v_c = (1.04)(2.06 \tfrac{m}{s}) = 2.14 \tfrac{m}{s}$$

AP-C Objectives (from College Board Learning Objectives for AP Physics)

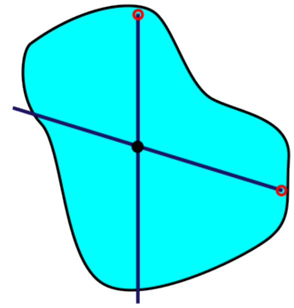
1. Center of Mass
   - a. Identify by inspection the center of mass of a symmetrical object.
   - b. Locate the center of mass of a system consisting of two such objects.

Center of Mass

Real objects are more complex than simple particles. You can treat an entire object as if its entire mass were contained at a single point, known as the object's **center of mass**. Alternately, the center of mass is the weighted average of the location of mass in an object.

Center of Mass by Inspection

1. For uniform density objects, the center of mass is the geometric center.
2. For objects with multiple parts, find the center of mass of each part and treat it as a point.
3. For irregular objects, suspend the object from two or more points and drop a plumb line. The intersection of those points is the center of mass.



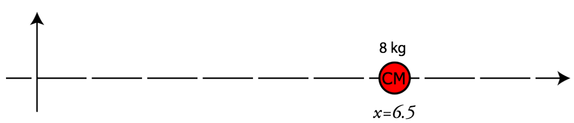
Calculating Center of Mass for a System of Particles

$$r_{CM}=\frac{\sum m\vec{r}}{\sum m} \qquad x_{CM}=\frac{m_1x_1+m_2x_2+\ldots}{m_1+m_2+\ldots} \qquad y_{CM}=\frac{m_1y_1+m_2y_2+\ldots}{m_1+m_2+\ldots}$$

Sample Problem

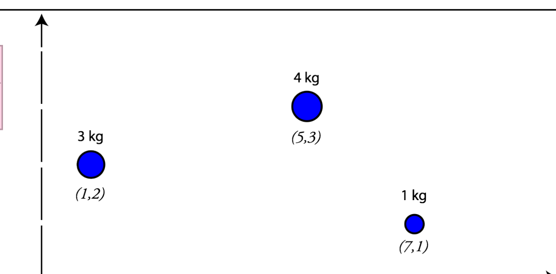
Find the center of mass of an object modeled as two separate masses on the x-axis. The first mass is 2 kg at an x-coordinate of 2 and the second mass is 6 kg at an x-coordinate of 8.

$$x_{CM}=\frac{m_1x_1+m_2x_2+\ldots}{m_1+m_2+\ldots}=\frac{(2kg)(2)+(6kg)(8)}{(2kg+6kg)}=6.5$$



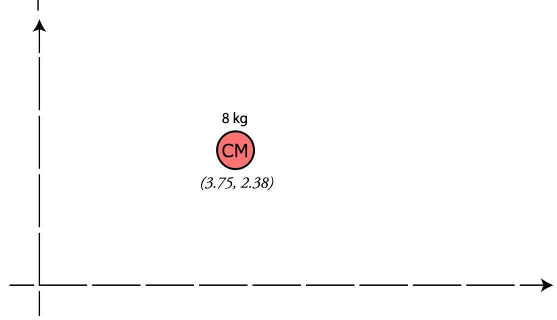
Sample Problem

Find the coordinates of the center of mass for the system shown.



$$x_{CM}=\frac{m_1x_1+m_2x_2+\ldots}{m_1+m_2+\ldots}=\frac{(3kg)(1)+(4kg)(5)+(1kg)(7)}{(3kg+4kg+1kg)}=3.75$$

$$y_{CM}=\frac{m_1y_1+m_2y_2+\ldots}{m_1+m_2+\ldots}=\frac{(3kg)(2)+(4kg)(3)+(1kg)(1)}{(3kg+4kg+1kg)}=2.38$$



Center of Mass by Integration

AP-C Objectives (from College Board Learning Objectives for AP Physics)

1. Center of Mass
   - a. Use integration to find the center of mass of a thin rod of non-uniform density.

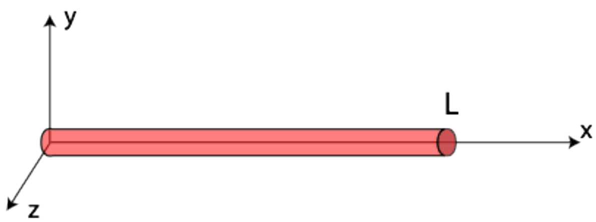
Finding Center of Mass by Integration

For more complex objects, you can find the center of mass by summing up all the little pieces of position vectors multiplied by the differential of mass and then dividing by the total mass.

$$\vec{r}_{CM} = \frac{\int \vec{r}\, dm}{M}$$

Sample Problem

Find the center of mass of a uniform rod of length L and mass M.



Strategy

Mass is evenly distributed from x=0 to x=L. dm/dx is mass per length (λ), so dm/dx=λ and therefore dm=λ dx

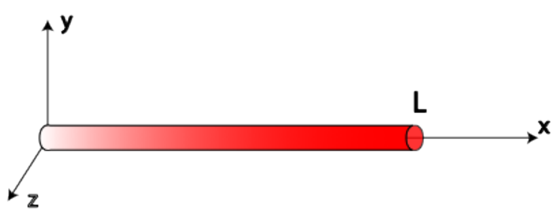
$$\vec{r}_{CM} = \frac{\int \vec{r}\, dm}{M} \xrightarrow{\vec{r}=x} \vec{r}_{CM} = \frac{\int x\, dm}{M} \xrightarrow{dm=\lambda dx} \vec{r}_{CM} = \frac{\int x\lambda\, dx}{M}$$

$$\vec{r}_{CM} = \frac{\lambda}{M}\int_{x=0}^{x=L} x\, dx = \frac{\lambda}{M}\frac{x^2}{2}\bigg|_{x=0}^{x=L} = \frac{\lambda}{M}\left(\frac{L^2}{2} - \frac{0^2}{2}\right) \rightarrow$$

$$\vec{r}_{CM} = \frac{\lambda L^2}{2M} \xrightarrow{M=\lambda L} \vec{r}_{CM} = \frac{\lambda L^2}{2\lambda L} \rightarrow \vec{r}_{CM} = \frac{L}{2}$$

Sample Problem

Find the center of mass of a non-uniform rod of length L and mass M whose density is given by λ=kx.



Strategy

Mass is non-uniformly distributed from x=0 to x=L with λ=kx, so dm=λdx=(kx)dx.

$$\vec{r}_{CM} = \frac{\int \vec{r}\, dm}{M} \xrightarrow{\vec{r}=x} \vec{r}_{CM} = \frac{\int x\, dm}{M} \xrightarrow[dm=(kx)dx]{\lambda=kx} \vec{r}_{CM} = \frac{\int x(kx)\, dx}{M}$$

$$\vec{r}_{CM} = \frac{k}{M}\int_{x=0}^{x=L} x^2\, dx = \frac{k}{M}\frac{x^3}{3}\bigg|_{x=0}^{x=L} = \frac{k}{M}\left(\frac{L^3}{3} - \frac{0^3}{3}\right) \rightarrow$$

$$\vec{r}_{CM} = \frac{kL^3}{3M} \xrightarrow{M=kL^2/2} \vec{r}_{CM} = \frac{2kL^3}{kL^2 3} \rightarrow \vec{r}_{CM} = \frac{2L}{3}$$

Find Total Mass

Find the total mass M by integrating the density from 0 to L for all little pieces of rod dx.

$$M = \int_{x=0}^{x=L} \lambda\, dx \xrightarrow{\lambda=kx} M = \int_{x=0}^{x=L} kx\, dx = k\int_{x=0}^{x=L} x\, dx \rightarrow$$

$$M = k\frac{x^2}{2}\bigg|_0^L = \frac{kL^2}{2}$$

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   - a. Apply the relation between center-of-mass velocity and linear momentum, and between center-of-mass acceleration and net external force for a system of particles.
   - b. Define center of gravity and use this concept to express the gravitational potential energy of a rigid object in terms of the position of its center of mass.

$$\vec{r}_{CM} = \frac{1}{M}\sum_i m_i\vec{r}_i \xrightarrow{\text{take derivatives}} \vec{v}_{cm} = \frac{1}{M}\sum_i m_i\vec{v}_i \xrightarrow{\vec{p}=m\vec{v}}$$

$$\vec{v}_{CM} = \frac{1}{M}\sum_i \vec{p}_i \to \sum_i \vec{p}_i = \vec{p}_{total} = M\vec{v}_{CM}$$

$$\vec{p}_{total} = M\vec{v}_{CM} \xrightarrow{F=\frac{d\vec{p}}{dt}} \frac{d\vec{p}_{total}}{dt} = \vec{F}_{total} = \frac{d}{dt}M\vec{v}_{CM} \to$$

$$\frac{d\vec{p}_{total}}{dt} = \vec{F}_{total} = M\frac{d}{dt}\vec{v}_{CM} = M\vec{a}_{CM}$$

Newton's 2nd Law!

Center of Gravity

1. Center of Gravity refers to the location at which the force of gravity acts upon an object as if it were a point particle with all its mass focused at that point.
   - a. In a uniform gravitational field, Center of Gravity and Center of Mass are the same.
   - b. In a non-uniform gravitational field, they may be different.