AP-C Objectives (from College Board Learning Objectives for AP Physics)

▼ 1. Simple Harmonic Motion

- a. Sketch or identify a graph of displacement as a function of time, and determine from such a graph the amplitude, period and frequency of the motion.
- b. Write down an appropriate expression for displacement of the form A sin **w**t or A cos **w**t to describe the motion.
- c. Find an expression for velocity as a function of time.
- d. State the relations between acceleration, velocity and displacement, and identify points in the motion where these quantities are zero or achieve their greatest positive and negative values.
- e. State and apply the relation between frequency and period.
- f. Recognize that a system that obeys a differential equation of the form $d^2x/dt^2 = -\omega^2 x$ must execute simple harmonic motion, and determine the frequency and period of such motion.
- g. State how the total energy of an oscillating system depends on the amplitude of the motion, sketch, or identify a graph of kinetic or potential energy as a function of time, and identify points in the motion where this energy is all potential or all kinetic.
- h. Calculate the kinetic and potential energies of an oscillating system as functions of time, sketch or identify graphs of these functions, and prove that the sum of kinetic and potential energy is constant.
- i. Calculate the maximum displacement or velocity of a particle that moves in simple harmonic motion with specified initial position and velocity.
- j. Develop a qualitative understanding of resonance so they can identify situations in which a system will resonate in response to a sinusoidal external force.

▼ 2. Mass on a Spring

- a. Derive the expression for the period of oscillation of a mass on a spring.
- b. Apply the expression for the period of oscillation of a mass on a spring.
- c. Analyze problems in which a mass hangs from a spring and oscillates vertically.
- d. Analyze problems in which a mass attached to a spring oscillates horizontally.
- e. Determine the period of oscillation for systems involving series or parallel combinations of identical springs, or springs of differing lengths.

▼ 3. Pendulums and Other Oscillations

- a. Derive the expression for the period of a simple pendulum.
- b. Apply the expression for the period of a simple pendulum.
- c. State what approximation must be made in deriving the period.
- d. Analyze the motion of a physical pendulum in order to determine the period of small oscillations.

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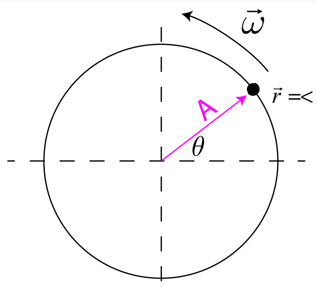
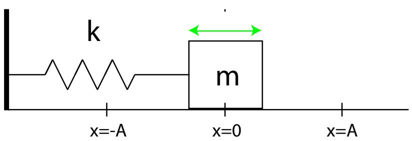
1. Simple Harmonic Motion
   - a. Sketch or identify a graph of displacement as a function of time, and determine from such a graph the amplitude, period and frequency of the motion.
   - b. Write down an appropriate expression for displacement of the form A sin ωt or A cos ωt to describe the motion.
   - c. Find an expression for velocity as a function of time.
   - d. State the relations between acceleration, velocity and displacement, and identify points in the motion where these quantities are zero or achieve their greatest positive and negative values.
   - e. State and apply the relation between frequency and period.
   - f. Recognize that a system that obeys a differential equation of the form $d^2x/dt^2 = -\omega^2 x$ must execute simple harmonic motion, and determine the frequency and period of such motion.
   - g. Calculate the maximum displacement or velocity of a particle that moves in simple harmonic motion with specified initial position and velocity.
   - h. State how the total energy of an oscillating system depends on the amplitude of the motion; sketch or identify a graph of kinetic or potential energy; and identify points where this energy is all potential or all kinetic.
   - i. Calculate the kinetic and potential energies of an oscillating system as functions of time.



$\vec{r} =< A\cos\theta, A\sin\theta > \longrightarrow \vec{r} =< A\cos(\omega t), A\sin(\omega t) >$

$$\omega = \frac{d\theta}{dt} \rightarrow d\theta = \omega dt \rightarrow \int_{\theta=0}^{\theta} d\theta = \int_{t=0}^{t} \omega\, dt \xrightarrow{\omega \text{ constant}}$$

$$\theta = \omega t$$



$$x = A\cos\theta \xrightarrow{\theta=\omega t} x(t) = A\cos(\omega t)$$

$$v = \frac{dx}{dt} = \frac{d}{dt}(A\cos(\omega t)) = -\omega A\sin(\omega t)$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{d}{dt}(-\omega A\sin(\omega t)) = -\omega^2 A\cos(\omega t)$$

$$x_{max} = A \quad v_{max} = \omega A \quad a_{max} = \omega^2 A$$

$$F = ma = -kx \xrightarrow{a=\frac{d^2x}{dt^2}} m\frac{d^2x}{dt^2} = -kx \rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$x(t) = A\cos(\omega t + \phi) \qquad \omega = \sqrt{\tfrac{k}{m}}$$

Period

Period (T) is the time for one cycle or complete revolution. Units are seconds [s].

$$T = \frac{1}{f}$$

Frequency

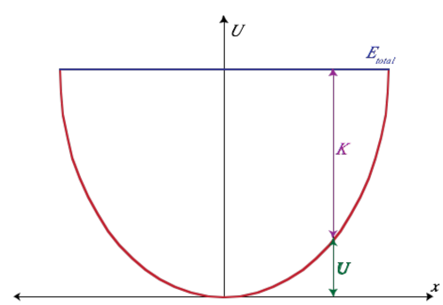
Frequency (f) is the number of cycles or revolutions per second. Units are 1/seconds, or Hertz [Hz].

Angular Frequency

Angular frequency (ω) is the number of radians per second, and it corresponds to an angular velocity for an object traveling in uniform circular motion.

$$\omega = 2\pi f = \frac{2\pi}{T}$$

General Form of SHM: $\frac{d^2x}{dt^2} + \omega^2 x = 0$ Solution: $x(t) = A\cos(\omega t + \phi)$



Energy of SHM

When an object undergoes SHM, kinetic and potential energy both vary with time, although total energy (E=K+U) remains constant.

Spring Example, where F=-kx

$$E = K + U = \tfrac{1}{2}kA^2\sin^2(\omega t) + \tfrac{1}{2}kA^2\cos^2(\omega t) = \tfrac{1}{2}kA^2\left[\sin^2(\omega t) + \cos^2(\omega t)\right] \rightarrow E = \tfrac{1}{2}kA^2$$

$$v(t) = \frac{dx}{dt} = -\omega A\sin(\omega t) \rightarrow K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left[-\omega A\sin(\omega t)\right]^2 \rightarrow$$

$$K = \tfrac{1}{2}m\omega^2 A^2\sin^2(\omega t) \xrightarrow{\omega^2 = k/m}$$

$$K = \tfrac{1}{2}kA^2\sin^2(\omega t)$$

$$W = \int_x^0 \vec{F}_x \bullet d\vec{x} = \int_0^x -kx\,dx = -k\left.\frac{x^2}{2}\right|_x^0 \rightarrow U_s = \tfrac{1}{2}kx^2$$

$$x(t) = A\cos(\omega t) \rightarrow U_s = \tfrac{1}{2}k\left[A\cos(\omega t)\right]^2 \rightarrow$$

$$U = \tfrac{1}{2}kA^2\cos^2(\omega t)$$

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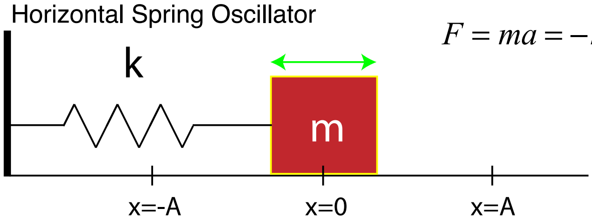
Horizontal Spring Oscillator



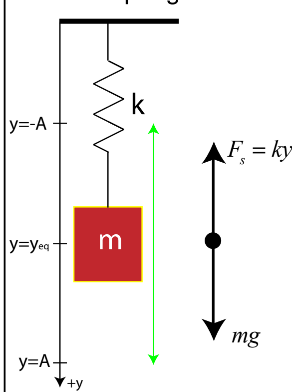
$$F = ma = -kx \xrightarrow{a=\frac{d^2x}{dt^2}} m\frac{d^2x}{dt^2} = -kx \to \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$x(t) = A\cos(\omega t + \phi) \qquad \omega = \sqrt{\frac{k}{m}}$$

$$\boxed{T = \frac{2\pi}{\omega} \xrightarrow{\omega=\sqrt{\frac{k}{m}}} T = 2\pi\sqrt{\frac{m}{k}}}$$

Vertical Spring Oscillator

$$F_{net_Y} = mg - ky = ma_y \xrightarrow{equilibrium} mg - ky = 0 \to y_{eq} = \frac{mg}{k}$$ at equilibrium



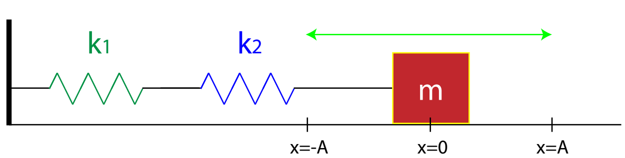
Once the system has settled at equilibrium, you set the mass in motion by pulling it some amount to either +A or -A. The new system can be analyzed as follows:

$$F_{net_Y} = mg - k(y_{eq} + A) = mg - ky_{eq} - kA \xrightarrow{mg-ky_{eq}=0} F_{net_Y} = -kA$$

This is the same analysis you would do for a horizontal spring system with spring constant k displaced an amount A from its equilibrium position. This means, in short, that to analyze a vertical spring system, all you do is find the new equilibrium position of the system taking into account the effect of gravity, then treat it as a system with only the spring force to deal with, oscillating around the new equilibrium point. No need to continue to deal with the force of gravity!

▼ Springs in Series

For springs in series, we'll calculate an equivalent spring constant for the system, then treat the system as if it's a single-spring system. To begin our analysis, we must realize the force on each spring must be the same by Newton's 3rd Law.



$$F = -k_1x_1 = -k_2x_2 \to x_1 = \frac{k_2}{k_1}x_2$$

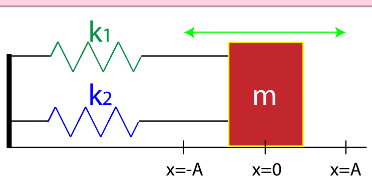
$$F = -k_{eq}(x_1 + x_2) \xrightarrow{x_1=\frac{k_2}{k_1}x_2} F = -k_{eq}(\frac{k_2}{k_1}x_2 + x_2)$$

$$\xrightarrow{F=-k_2x_2} k_2x_2 = k_{eq}x_2(\frac{k_2}{k_1}+1) \to k_2 = k_{eq}(\frac{k_2}{k_1}+1) \to$$

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

▼ Springs in Parallel

For springs in series, we'll calculate an equivalent spring constant for the system, then treat the system as if it's a single-spring system.



$$F = k_1x + k_2x = (k_1 + k_2)x = k_{eq}x \to$$

$$k_{eq} = k_1 + k_2$$

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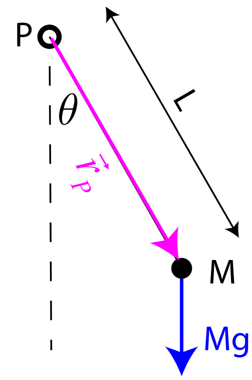
- d. Analyze the motion of a physical pendulum in order to determine the period of small oscillations.

▼ Period of a Simple Pendulum

Find the period of a mass M hanging from a light string from point P when oscillated through a small angle θ.



$$\vec{\tau}_P = \vec{r} \times \vec{F} = \vec{r}_P \times M\vec{g} \rightarrow \left|\vec{\tau}_P\right| = MgL\sin\theta \xrightarrow[\substack{\alpha \text{ negative since it points} \\ \text{opposite direction of } \theta}]{\tau_{net} = I\alpha}$$

$$-MgL\sin\theta = I_p\alpha \xrightarrow[\theta \text{ small}]{\sin\theta \approx \theta} -MgL\theta = I_p\alpha \xrightarrow{\alpha = d^2\theta / dt^2}$$

$$-MgL\theta = I_p\frac{d^2\theta}{dt^2} \rightarrow \frac{d^2\theta}{dt^2} + \frac{MgL}{I_P}\theta = 0$$

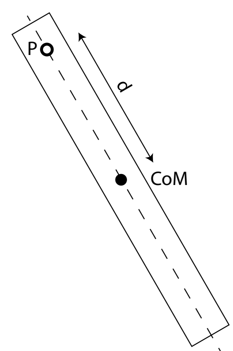
$$\theta(t) = A\cos(\omega t)$$

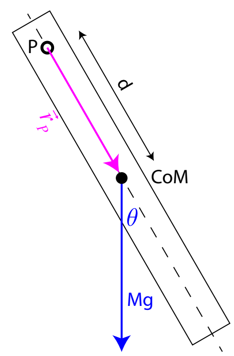
$$\omega^2 = \frac{MgL}{I_P} \rightarrow \omega = \sqrt{\frac{MgL}{I_P}}$$

$$\boxed{T = \frac{2\pi}{\omega} \xrightarrow{\omega = \sqrt{\frac{MgL}{I_P}}} T = 2\pi\sqrt{\frac{I_P}{MgL}} \xrightarrow{I_P = ML^2} T = 2\pi\sqrt{\frac{ML^2}{MgL}} \rightarrow T = 2\pi\sqrt{\frac{L}{g}}}$$

▼ Period of a Physical Pendulum

Find the period of a rod of mass M hanging from point P when oscillated through a small angle θ.





$$\vec{\tau}_{net} = \vec{r} \times \vec{F} = \vec{r}_p \times M\vec{g} \rightarrow \left|\vec{\tau}_{net}\right| = Mgd\sin\theta \xrightarrow[\substack{\alpha \text{ negative since it points} \\ \text{opposite direction of } \theta}]{\tau_{net} = I\alpha}$$

$$-Mgd\sin\theta = I_p\alpha \xrightarrow[\theta \text{ small}]{\sin\theta \approx \theta} -Mgd\theta = I_p\alpha \xrightarrow{\alpha = d^2\theta / dt^2}$$

$$-Mgd\theta = I_p\frac{d^2\theta}{dt^2} \rightarrow \frac{d^2\theta}{dt^2} + \frac{Mgd}{I_P}\theta = 0$$

Simple Harmonic Motion

Solution: $\theta(t) = A\cos(\omega t)$ $\quad \omega^2 = \frac{Mgd}{I_P} \rightarrow \omega = \sqrt{\frac{Mgd}{I_P}}$

Period: $T = \frac{2\pi}{\omega} \xrightarrow{\omega = \sqrt{\frac{Mgd}{I_P}}} T = 2\pi\sqrt{\frac{I_P}{Mgd}}$

$$\boxed{T = 2\pi\sqrt{\frac{\frac{ML^2}{12} + Md^2}{Mgd}} = 2\pi\sqrt{\frac{\frac{L^2}{12} + d^2}{gd}} = 2\pi\sqrt{\frac{L^2 + 12d^2}{12gd}}}$$

▼ Moment of Inertia (use Parallel Axis Theorem)

Having previously determined the moment of inertia of a solid rod about its center of mass is $ML^2/12$, we can use the PAT to determine the moment of inertia about point P located a distance d from the center of mass.

$$I_P = I_{CoM} + Md^2 = \frac{ML^2}{12} + Md^2$$